thinkplus

FORMULA BOOK

Number System

1. **Natural Numbers:** Set of counting numbers is called natural numbers. It is denoted by N. where, N = {1, 2, 3, ... ∞}
2. **Even Numbers:** The set of all natural numbers which are divisible by 2 are called even numbers. It is denoted by E. Where, E = {2, 4, 6, 8, 10, ... ∞}
3. **Odd Numbers:** The set of all natural numbers which are not divisible by 2 are called odd numbers. In other words, the natural numbers which are not even numbers, are odd numbers. i.e., O= {1, 3, 5, 7, ... ∞}
4. **Whole Numbers:** When zero is included in the set of natural numbers, then it forms set of whole numbers. It is denoted by W. Where, W = {0, 1, 2, 3,...... ∞}
5. **Integers:** When in the set of whole numbers, natural numbers with negative sign are included, then it becomes set of integers. It is denoted by I or Z.
   I = [- ∞,…-4, -3, -2, -1, 0, 1, 2, 3, 4,….,∞]
   Integers can further be classified into negative or positive Integers. Negative Integers are denoted by Z and positive Integers are denoted by Z^+.
   Z^- {-∞,....., -3, -2, -1} and
   Z^+ = {1, 2, 3, ... ∞}
   Further, 0 is neither negative nor positive integer.
6. **Prime Numbers:** The natural numbers which have no factors other than 1 and itself are called prime numbers. Note that,

   (i) In other words, they can be divided only by themselves or 1 only. As, 2, 3, 5, 7, 11 etc.

   (ii) All prime numbers other than 2 are odd numbers but all odd numbers are not prime numbers.

   (iii) Every prime number other than 2, 3 can be represented in the form of $6k \pm 1$. But every number that is this form may or may not be a prime number

7. **Co-Prime Numbers:** Two numbers which have no common factor except 1, are called Co-Prime numbers. Such as, 9 and 16, 4 and 17, 80 and 81 etc.
   It is not necessary that two co-prime numbers are prime always. They may or may not be prime numbers.
8. **Divisible numbers/Composite numbers:** The whole numbers which are divisible by numbers other than itself and 1 are called divisible numbers or we can say the numbers which are not prime numbers are composite or divisible numbers. As, 4, 6, 9, 15.
   Note: 1 is neither Prime number nor composite number. Composite numbers may be even or odd.
9. **Rational Numbers:** The numbers which can be expressed in the form of $\frac{p}{q}$where p and q are integers q and r co-primes and q $\neq$ 0 are called rational numbers. It is denoted by Q. These may be positive, or negative.

   e.g.$\frac{4}{5}, \frac{5}{1}, -\frac{1}{2}$ etc are rational numbers.

10. **Irrational Numbers:** The numbers which are not rational numbers, are called irrational numbers. Such as $\sqrt{2}$ = 1.414213562.......... , π= 3.141592653

11. **Real Numbers:** Set of all rational numbers as well as irrational numbers is called Real numbers. The square of all of them is positive.

12. **Cyclic Numbers:** Cyclic numbers are those numbers of n digits which when multiplied by any other number up to n gives same digits in a different order. They are in the same line.
As 142857
2 × 142857 = 285714
3 × 142857 = 428571
4 × 142857 = 571428
5 × 142857 = 714285

13. **Perfect Numbers:** If the sum of all divisors of a number N (except N) is equal to the number N itself then the number is called perfect number. Such as, 6, 28, 496. 8128 etc.
The factor of 6 are 1, 2 and 3
Since,

Perfect Number	**Sum of factors**
6	1 + 2 + 3 = 6
28	1 + 2 + 4 + 7 + 14 = 28
496	1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496
8128	1 + 2 + 4 + 8 + 16 + 32 + 64 + 127+ 254 + 508 + 1016 + 2032 + 4064 = 8128

14. In a perfect number, the sum of inverse of all of its factors including itself is 2 always.
e.g. Factors of 28 are 1,2,4,7,14 are $\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}=\frac{56}{28}=2$

15. **Complex Numbers:** Z = a + ib is called complex number, where a and b are real numbers, b $\neq$ 0 and i = √- 1. Such as, √-2. √3 etc.
So, a + ib or 4 + 5i are complex numbers.

Factors:-

16. The numbers with two factors are prime numbers
17. The numbers with three factors are squares of prime numbers
18. The numbers with four factors are cubes of prime numbers or product of two prime numbers
19. The numbers with five factors are fourth powers of prime numbers.
20. Perfect squares have odd number of factors
21. Number after being expressed as N = $a^m \times b^n \times c^o \times d^p \times$ …(prime Factorization) where a, b, c, d …are prime factors of N, and m, n, o, p … are their powers
Then the number of factors will be: $(m + 1)(n + 1)(o + 1)(p + 1)$….
22. Number of even factors $= m(n+1)(o+1)(p+1)$ … .. (Here, m is the power of even prime factors and $n, o, p,$ …are powers of odd prime factors)

23. Number of odd factors = $(n+1)(o+1)(p+1)$ …. (Here, $n, o, p,$…. are powers of odd prime factors of the number)

24. Sum of all the factors = $(a^0 + a^1 + \ldots + a^m)(b^1 + b^2 + \ldots + b^n)(c^1 + c^2 + \ldots + c^o)(d^1 + d^2 + \ldots + d^p)$…..
Note:- From here we can use the sum of terms in a Geometric progressions to get the sum of terms inside each bracket.

25. Sum of all the even factors of a number =
$(a^1 + \ldots + a^m)(b^1 + b^2 + \ldots + b^n)(c^1 + c^2 + \ldots + c^o)(d^1 + d^2 + \ldots + d^p)$….. ; where a is the even prime factor of the number and $b, c, d,$ …. are odd prime factors of the number and $m, n, o, p, \ldots$ are their powers respectively

26. Sum of all odd factors of a number = $(b^1+b^2+\ldots+b^n)(c^1+c^2+\ldots+c^o)(d^1+d^2+\ldots+d^p)$….. where $b, c, d,$.. are odd prime factors of the number and $n, o, p,$.. are their powers respectively.

27. Number of ways in which the number can be represented as a product of two distinct factors:
(i) Perfect squares: - $\frac{Number\ of\ factors - 1}{2}$;
$\frac{Number\ of\ factors + 1}{2}$, if product of same factors is also included
(ii) Non-Perfect squares: - $\frac{Number\ of\ factors}{2}$

28. Product of all factors: - $(\text{Number})^{\text{Number of factors}/2}$

29. **Co-primes:** The number of co-primes to N which are less than N $= N\left(1 - \frac{1}{a}\right)\left(1 - \frac{1}{b}\right)\ldots$
a, b, are Prime Bases in the prime factorization of N.

30. **Sum of all the Co-primes of N which are less than N is:-** $\frac{Number\ of\ co\text{-}primes}{2} \times N$

31. Highest power of 'P' which can exactly divide n! is: $\left[\frac{n}{P}\right] + \left[\frac{n}{P^2}\right] + \left[\frac{n}{P^3}\right] + \ldots + \left[\frac{n}{P^x}\right]$.
this formula goes on till we get $n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - \ldots\ldots - {}^nC_{n-1}(1)^m = 1$
Note : [] is greatest integer function.

32. Number of zeroes in n! is equal to highest power of 5 in n!.

33. **Divisibility Rules: -**

Number	Divisibility Rule (DR)
2	Last digit is a multiple of 2
4	Last two digits is a multiple of 4
8	Last three digits is a multiple of 8
16	Last four digits is a multiple of 16
3	Sum of digits is a multiple of 3
9	Sum of digits is a multiple of 9
5	Unit digit should be 5 or 0
10	Unit digit is zero
6	DR of 3 and 2
12	DR of 4 and 3 (we cannot take 2 and 6, as they are not co-primes)
15	DR of 5 and 3

18	DR of 2 and 9 (we cannot take 6 and 3, as they are not co-primes)
11	Difference between two sum of Alternate digits is a multiple of 11 or zero

34. **Power Cyclicity to Find Unit Digit: -**

Unit Digit	Repeated digits	Power cyclicity
0	0	1
1	1	1
2	2, 4, 8, 6	4
3	3, 9, 7, 1	4
4	4, 6	2
5	5	1
6	6	1
7	7, 9, 3, 1	4
8	8, 4, 2, 6	4
9	9, 1	2

Last two-digits: -

35. Last two digits of $(\text{Odd})^{20m}$ = 01; m is a natural number (Exception 5)
36. Last two digits of a number with 5 is 75 only when ten's digit of the base is odd and also the power is odd, else the last two digits will be 25.
37. Last two digits any number N^2 is equal to the last two digits of $(50 - N)^2$, $(50 + N)^2$ and $(100-N)^2$
38. Last two digits of 2^{10m} = 24; if m is odd
39. Last two digits of 2^{10m} = 76; if m is even

Remainders:

40. $R\left[\frac{(N-1)!}{N}\right] = N - 1$, where N is a prime number
41. $R\left[\frac{(N-2)!}{N}\right] = 1$, where N is a prime number
42. $R\left[\frac{M^{N-1}}{N}\right] = 1$, Where N is a prime number, M and N are pair of Co-primes.
43. $R\left[\frac{M^{N}}{N}\right] = M$, Where N is a prime number, M and N are pair of Co-primes.
44. The expression $a^n - b^n$ is always divisible by a - b.
45. The expression $a^n - b^n$ is divisible by a + b only when n is odd.
46. The expression $a^n + b^n$ is divisible by a + b if n is odd. If n is even, it is not divisible by a + b.

HCF & LCM

Highest Common Factor (H.C.F): It is also called Greatest common Divisor (G.C.D). When a greatest number divides perfectly the two or more given numbers then that number is called the H.C.F. of two or more given numbers.

e.g. The H.C.F of 10, 20, 30 is 10 as they are perfectly divided by 10.5 and 2 and 10 is highest or greatest of them.

Least common Multiple (L.C.M.): The least number which is divisible by two or more given numbers, that least number is called L.C.M. of the numbers.
L.C.M. of 3,5,6 is 30, because all 3 numbers divide 30, 60, 90, and so on perfectly and 30 is minimum of them.

Factor and Multiple: If a number m divides perfectly second number n. then m is called the factor of n and n is called the multiple of m.

47. 1^{st} number x 2^{nd} number = L.C.M. x H.C.F.
48. If the ratio of two numbers is a : b, (lowest form) then Numbers are ak and bk, where k is a constant and hence, H.C.F. is k and L.C.M. is abk.
49. LCM of fractions = $\frac{\text{LCMofnumerators}}{\text{HCFofDenominators}}$
50. HCF of fractions = $\frac{\text{HCFofnumerators}}{\text{LCM ofDenominators}}$
51. If there is no common factor between two numbers, then L.C.M. will be the product of both numbers.
52. If there are 'n' numbers in a set and H.C.F. of any two numbers is H.C.F. and L.C.M. of all 'n' numbers is 'L' then product of all 'n' numbers is $[(H)^{n-1} \times L]$
53. When a number is divided by a, b or c leaving same remainder 'r' in each case then that number must be in the form of Lk + r where L is LCM of a, b and c, and k is a natural number. The least possible number such is at k=1
54. When a number is divided by a, b or c leaving remainders p, q, or r respectively such that the difference between divisor and remainder in each case is same i.e., (a - p) = (b - q) = (c - r) = t (say) then that number must be in the form of Lk-t, where L is LCM of a, b and c and k is a natural number. The least possible such number is possible at k = 1.
55. The largest number which divides the numbers a, b and c the remainders are same then that largest number is given by H.C.F. of (a - b), (b - c) and (c - a).
56. The largest number which divides the numbers a, b and c give remainders as p, q, r respectively is given by H.C.F. of (a - p), (b - q) and (c - r).

Percentages

57. **Percentage = (Value ÷ Total) × 100**
    This formula is used to calculate the percentage of one quantity with respect to the total quantity.
58. **Value = (Percentage ÷ 100) × Total**
    This formula is used to find out the part of the quantity when the percentage and the whole quantity is given.
59. **Total = Value ÷ (Percentage ÷ 100)**
    This formula is used to find out the total quantity when the percentage and part of the quantity is given.

60. **Percent increase = [(New value - Old value) ÷ Old value] × 100**
This formula is used to find out the percentage increase between two values.

61. **Percent decrease = [(Old value - New value) ÷ Old value] × 100**
This formula is used to find out the percentage decrease between two values.

62. **Percentage-Fraction Conversion Table:**

DENOMINATOR	NUMERATOR											
	1	2	3	4	5	6	7	8	9	10	11	12
1	100%	200%	300%	400%	500%	600%	700%	800%	900%	1000%	1100%	1200%
2	50%	100%	150%	200%	250%	300%	350%	400%	450%	500%	550%	600%
3	33.33%	66.67%	100%	133.33%	166.67%	200%	233.33%	266.67%	300%	333.33%	366.67%	400%
4	25%	50%	75%	100%	125%	150%	175%	200%	225%	250%	275%	300%
5	20%	40%	60%	80%	100%	120%	140%	160%	180%	200%	220%	240%
6	16.66%	33.33%	50.00%	66.66%	83.33%	100.00%	116.66%	133.33%	150.00%	166.66%	183.33%	200.00%
7	14.28%	28.56%	42.85%	57.13%	71.42%	85.71%	100.00%	114.28%	128.56%	142.85%	157.13%	171.42%
8	12.50%	25%	37.50%	50%	62.50%	75%	87.50%	100%	112.50%	125%	137.50%	150%
9	11.11%	22.22%	33.33%	44.44%	55.55%	66.66%	77.77%	88.88%	99.99%	111.11%	122.22%	133.33%
10	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%	110%	120%
11	9.09%	18.18%	27.27%	36.36%	45.45%	54.54%	63.63%	72.72%	81.81%	90.90%	100.00%	109.09%
12	8.33%	16.66%	25%	33.33%	41.67%	50%	58.33%	66.66%	75%	83.33%	91.66%	100%
15	6.66%	13.33%	20%	26.66%	33.33%	40%	46.66%	53.33%	60%	66.66%	73.33%	80%

63. 100% of a number is always the **number itself**

64. 10% of a number is **0.1 of the same number**

65. 1% of a number is **0.01 of the same number**

66. If there is a percentage increase and you want to return to the original value:
Let's say something increases by a fraction x/y (for example, a 25% increase would be 1/4).
To bring it back down to the original value, you need to decrease the new value by:
(100 × x) / (y + x) percent.

67. If there is a percentage decrease and you want to return to the original value:
Let's say something decreases by a fraction x/y (like a 20% decrease would be 1/5).
To bring it back up to the original value, you need to increase the reduced value by:
(100 × x) / (y − x) percent.
(Note: This only works if y > x)
Note:
This concept can be used in product constancy. If one of the two factors increases/decreases, The other factor decreases/increases according to the concept mentioned above.

68. **Whenever there is "a" percent increase in the value and again there is "a" percent increase in the value, then the effective increase in the percentage will be:** $\left[+a+a+{\frac{a}{100}}^{2}\right]\%$

69. **Whenever there is "a" percent increase in the value and again there is "a" percent decrease in the value, then the effective change in the percentage will be:** $\left[{\frac{a}{100}}^{2}\right]\%$ **decrease**

70. **Whenever there is "a" percent increase in the value and again there is "b" percent increase in the value, then the effective increase in the percentage will be:** $\left[+a+b+\frac{ab}{100}\right]\%$

71. **Whenever there is "a" percent increase in the value and again there is "b" percent decrease in the value, then the effective change in the percentage will be:** $\left[+a-b-\frac{ab}{100}\right]\%$

72. **Whenever there is "a" percent increase/decrease in the value and again there is "b" percent decrease/increase in the value, then the effective increase in the percentage will be:**
$\left[\pm a\pm b\pm\frac{ab}{100}\right]\%$
Note: The first two symbols (in the formula) depend on the increase/decrease. If the value is increasing, we use "+" and in case of a decrease we use "-".
The third symbol depend on the previous operations.. it will be the multiplication of the previous operations
i) in case of both increase, it will be "+".
ii) in case of both decrease, it will be "+".
iii) in case first the value is increased and then decreased, it will be "-".
iv) in case first the value is decreased and then increased, it will be "-".

Profit, Loss, and Discount

73. **Cost Price (CP)**: The basic price of an item/article
74. **Selling Price (SP)**: The price at which the item/article is sold.
75. **Profit:** Whenever the selling price is **more** than the cost price, there will be a profit.

Profit = SP – CP

76. **Loss:** Whenever the Selling Price is less than the Cost Price, there will be a Loss.

Loss = CP – SP

77. **Profit/Loss Percentage:**
i) Profit / Loss Percentage is always calculated on Cost Price.
ii) Profit% = $\frac{SP-CP}{CP}\times(100)$
iii) Loss% = $\frac{CP-SP}{CP}\times(100)$

78. **Selling Price:**
i) **SP = CP + Profit (or) SP = CP – Loss**
ii) If an article is sold at x% profit, then **SP = CP**$\left(\frac{100+x}{100}\right)$
iii) If an article is sold at x% loss, then **SP = CP**$\left(\frac{100-x}{100}\right)$

79. If the selling price of two objects is similar objects, one at a loss of x% and another at a gain of x%, then there will always be a loss of $\left(\frac{x^2}{100}\right)\%$

80. If a dishonest trader sells his goods at Cost price but uses false weight, then his profit % is:

Profit % = (True Weight – False Weight) ÷ (False Weight) × 100

(or)

Profit% = (Quantity Left) ÷ (Quantity Sold) × 100

81. **Marked Price (MP):**
    i) The price at which an article/item is intended to be sold.
    ii) If an article is marked up by x%, then **MP = CP** $\left(\frac{100+x}{100}\right)$
82. **Discount** = Marked price – Selling Price
83. **Discount Percentage:** $\frac{Discount}{MP} \times 100$
    Important Note: Discount is always calculated on Marked Price, not on Cost Price or Selling Price.
84. **Selling price:**
    If an article/item is sold after allowing x% discount, then **the Selling Price is =** $MP\left(\frac{100-x}{100}\right)$
85. **Successive Discounts:** When an article is offered x% discount and again offered a discount by y% then the successive
    discount is
    $\left(-x-y+\frac{xy}{100}\right)$% (or) $\left(x+y-\frac{xy}{100}\right)$%

Simple Interest & Compound Interest

86. **Simple Interest (SI)** = $\frac{PTR}{100}$
    where: P = Principal (the initial amount of money), R = Rate of interest (in decimal),
    T = Time (in years)
    Funda: *Finding $TR\%$ of the total principal will give you the simple interest.*
    For example, $P = 1000, T = 3\ years, R = 5\%\ per\ annum$.
    The Simple Interest will be: (3×5) % of 1000. Which is 15% of 1000. Therefore, the Simple Interest will be 150 at the end of three years
87. Time Period: T= $\frac{SI\times100}{P\times R}$, Rate of Interest: R=$\frac{SI\times100}{P\times T}$, Principal= $\frac{SI\times100}{R\times T}$
88. The formula for finding the Total Amount (principal plus interest) is:
    Total Amount = P + Simple Interest = $P+\frac{PTR}{100}$
89. If a certain sum becomes 'n' times of itself under simple interest, then the rate percent per annum is:
    R= $\left(\frac{(n-1)}{T}\right)100\%$

    T= $\left(\frac{(n-1)}{R}\right)100\%$

90. **Compound Interest (CI)** = $P\left(\frac{100+R}{100}\right)^n - P$

where P = Principal, R = rate of interest, n = no. of years

91. Total Amount under compound Interest will be: $P\left(\frac{100+R}{100}\right)^n$

92. **Compound Interest when calculated Half yearly**: The rate of interest becomes **half (i.e. $\frac{R}{2}$%)** and number of times interest calculated gets **twice (i.e. 2n).**

93. **Compound Interest when calculated Quarterly:** The rate of interest becomes $\frac{R}{4}$%and number of the times interest calculated gets **four times (4n)**

94. **Compound Interest when calculated monthly:** The rate of interest becomes $\frac{R}{12}$% and number of times the interest calculated gets **four times (12n)**

95. **The difference between CI and SI on a sum 'P' in 2 years at the rate 'R%':**

$$CI - SI = P\left(\frac{R}{100}\right)^2$$

The Difference for 3 years: $CI - SI = P\left(\frac{R}{100}\right)^2 \times \left(3 + \frac{R}{100}\right)$

96. **Funda to Calculate Compound Interest is:**
Using Pascal Triangle:

1 ———————— 0 years
1 1 ———————— 1 year
1 2 1 ———————— 2 years
1 3 3 1 ———————— 3 years
1 4 6 4 1 ———————— 4 years
1 5 10 10 5 1 ———————— 5 years

For example:
If the Principal is Rs. 10,000, the rate of Interest is 5% and Interest compounded annually for 2 years. Find the Compound Interest:
Sol: Take 1 2 1 for 2 years

Code	Amount	Total	Type
1	10000	1 ×10000 =10000	Principal
2	5% of 10000 = 500	2 ×500 = 1000	interest
1	5% of 500 = 25	1 ×25 = 25	
		11025	**Total**

Total amount after two years is 11025. Excluding Principal, the interest amount is 1025
Note: To find the difference between SI and CI. Exclude the Principal and first line of Interest. Therefore, the difference is 25

If the Principal is Rs.10,000, the rate of Interest is 5% and Interest compounded annually for 3 years. Find the Compound
Interest:
Sol: Take 1 3 3 1 for 3 years

Code	Amount	Total	Type
1	10000	1 ×10000 = 10000	Principal
3	5% of 10000 = 500	3 ×500 = 1500	interest
3	5% of 500 = 25	3 ×25 = 75	
1	5% of 25= 1.25	1 ×1.25	
		11576.25	**Total**

Note: To find the difference between SI and CI compounded annually. Exclude the Principal and First row of Interest.

Difference= 3 ×25 + 1 ×1.25 = 76.25

97. If a sum becomes ‘n’ times itself in ‘t’ years on C.I, then r% = $\left[\left[n^{\frac{1}{t}} - 1\right] \times 100\right]$%

98. A certain sum at C.I becomes ‘m’ times in ‘t_1’ years and ‘n’ in ‘t_2’ years then: $m^{\frac{1}{t_1}} = n^{\frac{1}{t_2}}$

99. **Instalments under simple Interest: -**
Let ‘k’ be the amount of each instalment and ‘n be the number of instalments’, then$P + \frac{PTR}{100} = k + \left(k + \frac{kR}{100}\right) + \left(k + \frac{2kR}{100}\right) + \left(k + \frac{3kR}{100}\right) + \ldots + \left(k + \frac{nkR}{100}\right)$

100. **Instalment under compound Interest: -**
Let ‘k’ be the amount of each instalment and ‘n be the number of instalments’, then

$$P = \frac{k}{\left(1 + \frac{R}{100}\right)} + \frac{k}{\left(1 + \frac{R}{100}\right)^2} + \frac{k}{\left(1 + \frac{R}{100}\right)^3} + \ldots + \frac{k}{\left(1 + \frac{R}{100}\right)^n}$$

Averages

101. Averages of two or more quantities is
Average = Sum of observations / Total number of observations
102. The average of first ‘n’ consecutive natural numbers is: $\frac{n+1}{2}$
103. The average of first ‘n’ consecutive even natural numbers is: $n + 1$
104. The average of first ‘n’ consecutive odd natural numbers is: n
105. The average of squares of first ‘n’ consecutive natural numbers is: $\frac{(n+1)(2n+1)}{6}$
106. The average of cubes of first ‘n’ consecutive natural numbers is: $\frac{n(n+1)^2}{4}$
107. The average of certain consecutive terms or terms which are in Arithmetic Progression is:
$=\frac{1}{2}$ (First term + Last term)
108. Weighted Average: If the average of ‘n_1’ numbers is given as ‘A_1’ and that of ‘n_2’ is given as ‘A_2’ and that of ‘n_3’ is given as ‘A_3’ ……and that of ‘n_n’ Then the average or the weighted average will be:

Weighted Average= $\frac{\text{Total Sum}}{\text{Total Number}} = \frac{n_1A_1 + n_2A_2 + n_3A_3 + \ldots + n_nA_n}{n_1 + n_2 + n_3 + \ldots + n_n}$

109. If the value of each of the observation/quantity is increased or decreased or multiplied or divided by a number ‘k’ then: The average of all the observations/ quantities will also increase or decrease or multiply or divide respectively by ‘k’.
110. Mode = 3 Median - 2 Mean

Mixtures and Alligations

111. Suppose a container contains Liquids A and B. If certain amount from the container is removed and replaced with the same amount of liquid B and this operation being repeated for 'n ' times, then after n^th operation, final quantity of Liquid A will be: $F = I\left(\frac{I-R}{I}\right)^n$,

F= Final Quantity of liquid A
I = Initial Quantity of Liquid A
R = Quantity of liquid A removed
n = no. of times the operation is repeated

112. If X% of content is removed from a container containing Liquid A and Liquid B and replaced with liquid B and if this operation being repeated for 'n' times, then after nth operation, Final Quantity of liquid will be: $F = I\left(1 - \frac{X}{100}\right)^n$

113. If Q_1 and Q_2 quantities of two different qualities with average prices A and B respectively are mixed together then the overall average price of the mixture will be M such that $M = \frac{Q_1A+Q_2B}{Q_1+Q_2}$

114. If Q_1 and Q_2 quantities of two different qualities/prices A and B respectively are mixed together, and if M is the average Price of the mixture. Then the ratio in which the A and B are mixed in the mixture is given by: $\frac{Q_1}{Q_2} = \frac{B-M}{M-A}$

Assuming A is the price of the cheaper quality and B is the Better Quality and M is the Price of the Mixture after A and B are mixed in the ratio Q_1 and Q_2

115. **Alligation method:**

CP of Cheaper Quality (A) **CP of Dearer Quality (B)**

Average Price (M)

$B - M$ $M - A$

$$\frac{Q_1}{Q_2} = \frac{B-M}{M-A}$$

(Q_1 : Q_2 is the ratio in which the Quantities of Cheaper quality and Dearer Quality are mixed respectively in the mixture)

Ratio Proportion

116. **Mixed Ratios**: Let a : b and x:y be two ratios, then the mixed ratios will be: ax : by
117. **Duplicate ratio**: Duplicate Ratio of a : b is a^2 : b^2
118. **Sub-Duplicate ratios:** The sub-duplicate ratio of a : b is $\sqrt{a}:\sqrt{b}$
119. **Triplicate ratio**: The triplicate ratio of a : b is a^3 : b^3
120. **Sub-Triplicate ratio**: The sub-triplicate ratio of a : b is $\sqrt[3]{a}:\sqrt[3]{b}$
121. **Inverse ratio**: The inverse ratio of a : b is b : a

    The inverse ratio of a:b:c is **,** which is bc : ac : ab
122. If a : b : : c : d, then 'a' and 'd' are extremes and 'b' and 'c' are called means.
123. Products of extremes = product of means (i.e, ad = bc)
124. **Proportion:** a, b, c, d are said to be in proportion when a:b = c:d and can written as a : b : : c : d
125. **Invertendo of** a : b : : c : d is $b:a::d:c$
126. **Alternendo of a:b :: c:d is** $a:c::b:d$
127. **Componendo of a:b :: c:d is** $(a+b):b::(c+d):d$
128. **Dividendo of a:b :: c:d is** $(a-b):b::(c-d):d$
129. **Componendo and Dividendo of a:b :: c:d is** $(a+b):(a-b)::(c+d):(c-d)$
130. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots=k$, then: $\frac{a+c+e+\ldots}{b+d+f+\ldots}=k$ and also, $\frac{pa+qc+re+\ldots}{pb+qd+rf+\ldots}=k$
131. If x is the **mean proportion** between a and b, then**,** $a:x::x:b \quad \rightarrow x^2=ab$

    So the mean proportion of a and b Will be $x=\sqrt{ab}$
132. **Mean proportion between** $(x-a)$ **and** $(x-b)$ **is** $\sqrt{(x-a)(x-b)}$
133. **If** 'x' is the **third proportion** of a and b then, $a:b::b:x \quad \rightarrow x=\frac{b^2}{a}$
134. If 'x' is the **fourth proportion** of a, b and c, then $x:a::b:c \rightarrow x=\frac{bc}{a}$
135. If 'x' is the **first proportion** of a, b, and c , then $x:a::b:c \quad \rightarrow x=\frac{ab}{c}$
136. If partner A, partner B, partner C,… invested amount of a, b, c, ... (respectively) for the time period of p, q, r… months (respectively), then the ratio of profit shares will be:-

    ap : bq : cr : ….
137. If all the partners invest their money for same time period, then the ratio of profit shares will be in the ratio of their investments.
138. If $A\alpha B$ (A varies as B), then $\frac{A_1}{A_2}=\frac{B_1}{B_2}$
139. If $A\alpha\frac{1}{B}$ (A varies inversely as B), then $\frac{A_1}{A_2}=\frac{B_2}{B_1}$
140. If $A\alpha B$and $A\alpha C$, then $\frac{A_1}{A_2}=\frac{B_1}{B_2}\times\frac{C_1}{C_2}$
141. If $A\alpha B$and $A\alpha\frac{1}{C}$, then $\frac{A_1}{A_2}=\frac{B_1}{B_2}\times\frac{C_2}{C_1}$

Note: - A_1, A_2 are values of A corresponding to B_1, B_2, which are values of B.

Time and Work

142. The most basic concept in time and work is the fundamental formula that relates work, rate, and time. This formula is: **Work = Rate × Time**

Work (W) represents the total amount of work to be done.

Rate (R) / Efficiency represents the rate at which the work is being done.
It could be tasks completed per hour, items produced per day, etc.
The rate of work varies along with efficiencies or man power.

Time (T) represents the amount of time it takes to complete the work.

From this Formula, we can also get **Time** = $\frac{Work}{Rate}$

143. **If A can finish a piece of work in X days, then work done in one day is** $\frac{1}{x}$ of the total work.
If B can finish a piece of work in Y days, then work done in one is $\frac{1}{y}$ of the total work.
If A and B work together, they finish $\frac{1}{x}+\frac{1}{y}=\left(\frac{x+y}{xy}\right)$ of the total work in one day.
Therefore, working together, they take $\frac{xy}{x+y}$ to finish the entire work.

144. If A, B and C, working alone can finish a piece a work in x, y, z days respectively. Then working together, they can finish the work in $\frac{xyz}{xy+yz+xz}$

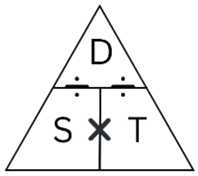
145. If the efficiencies of the workers is in the ratio a : b : c:….., then time taken by them to finish the work will be in the ratio:- $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$: …….. (With work being constant, the time is inversely proportional to efficiency)

146. If M_1 men can finish W_1 work in D_1 days working for H_1 hours each day and If M_2 men can finish W_2 work in D_2 days working for H_2 hours each day.
Then, $\frac{M_1D_1H_1}{W_1}=\frac{M_2D_2H_2}{W_2}$
Note: Can ignore H_1 hours and H_2 hours if it is not specified in the problem
Then, $\frac{M_1D_1}{W_1}=\frac{M_2D_2}{W_2}$
Note: If the work is same, then $M_1D_1=M_2D_2$

Time Speed and Distance

147. Speed is measured as the rate at which an object travels/ covers a distance per unit of time.

So, we can say that **Speed = Distance/Time**.
Now, by transposing **Speed = Distance/Time,** we get:
Distance = Speed $\times$ **Time** and
Time = Distance/Speed.



148. To convert km/h to m/s, The speed given in Km/h should be Multiplied with $\frac{5}{18}$ Because 1kmph is $\frac{5}{18}$m/s
149. To convert m/s to km/h, The speed given in m/s should be Multiplied with $\frac{18}{5}$Because 1m/s is$\frac{18}{5}$km/h
150. Average Speed= Total Distance / Total Time = $\frac{d_1+d_2+d_3+\ldots}{t_1+t_2+t_3+\ldots}$, where $d_1, d_2, d_3, \ldots$ are fractions of total distance which add up to total distance. $t_1, t_2, t_3, \ldots$ are time taken to cover $d_1, d_2, d_3, \ldots$ respectively.
Note: when speeds are given, we can write $t_1 = \frac{d_1}{s_1}, t_2 = \frac{d_2}{s_2}, \ldots\ldots$
151. Average speed when the journey is divided into 2 equal halves, and each half is covered with speeds S_1 and s_2 respectively is given by $\frac{2S_1S_2}{S_1+S_2}$
152. Average speed when the journey is divided into 3 equal halves, and each part is covered with speeds S_1, S_2 and S_3 respectively is given by $\frac{3S_1S_2S_3}{S_1S_2+S_2S_3+S_1S_3}$
153. If an object increases/ decreases it's speed to cover a distance in less/more time respectively, then

Distance = $\frac{s_1.s_2}{|s_1-s_2|} \times |t_1 - t_2|$, where $s_{1,}s_2$ refers to speeds and $t_{1,}t_2$ refers to respective times

Relative Speed:

154. When two objects travel in same direction, the relative speed will be the difference between the speeds of the objects: $S_1 - S_2$
155. When two objects travelling towards each other, the relative speed will be the sum of the speeds of the objects. $S_1 + S_2$
156. If two objects are separated by certain distance 'd'. Assuming that the faster object with the speed of S_1 is behind the slower one with the speed of S_2. While travelling in the same direction, they meet in: $\frac{d}{S_1-S_2}$
157. If two objects are separated by certain distance 'd',. started towards each other simultaneously towards each other, Then they meet after$\frac{d}{S_1+S_2}$

158. Two objects starting from two points A and B towards B and A with the speeds of S_1 and S_2 respectively. After meeting each other at a point, they reach their destinations in t_1 and t_2 times respectively. Then $\frac{S_1}{S_2} = \sqrt{\frac{t_2}{t_1}}$

159. If a train crosses a pole, man or something whose length is very negligible, then time taken to cross it is the length of the train(*l*) divided by the Speed of the train(s): $\frac{l}{s}$

160. If a train of length '*l*' crosses a stationary object (For eg. A platform, a Bridge, etc) whose length is given as 'm'. If the speed of the train is *s*. It crosses the platform in $\frac{l+m}{s}$

161. If length of two trains is t_1 and t_2, travelling at a speed of s_1 and s_2, one train to cross another, it takes $\frac{t_1+t_2}{s_1-s_2}$, when they are travelling in the same direction.

162. If length of two trains is t_1 and t_2, travelling at a speed of s_1 and s_2, one train to cross another, it takes $\frac{t_1+t_2}{s_1+s_2}$, when they are travelling in the opposite direction.

163. Excluding stoppage, the average speed of an object/vehicle be x and with stoppages, its average speed is y. Then:
Time the object has stopped in an hour: (Difference between their average speed) ÷ (Speed without Stoppage) = $\frac{x-y}{x}$

Boats and Streams: If the speed of the boat/swimmer in still water is B and the speed of the stream is S, then
The speed of the boat/swimmer in the direction of the stream (Downstream) = **(B + S) kmph or m/s**

164. ii) The speed of the boat/swimmer in the direction against the stream (Upstream) = **(B – S) kmph or m/s**

If the speed of the boat in upstream is 'U', and the speed of the boat in downstream is 'D', then,
The speed of the boat is: B = $\frac{D+U}{2}$

165. The speed of the stream is: S = $\frac{D-U}{2}$

Tracks and Races:

166. If A and B are two runners started simultaneously, they meet for the first time on the track after $t(AB) = \frac{l}{R_s}$, where l is length of the track and R_s is the relative speed. Relative speed is subject to the directions in which the runners are running.

167. When there are three runners, A, B, C. They all together meet on the track after $= LCM\ (AB\ Meeting\ time, AC\ Meeting\ time), (Assuming\ that\ A\ is\ the\ fastest)$
Note: - We can find the meeting time of AB and meeting time of AC using the previous formula

168. To meet at the starting point of the circular track, runners A, B, C… take $LCM\ (ta, tb, tc, \dots)$
t_a= time taken to finish a complete round by A…
t_b= time taken to finish a complete round by B…

169. When two bodies move around a circular track in the same direction with speeds in the ratios $a : b$, the maximum number of distinct meeting points = difference in the ratio= $a - b$

170. When two bodies move around a circular track in the opposite direction with speeds in the ratios $a : b$, the maximum number of distinct meeting points = sum of the ratio= $a + b$

171. In a linear race, the runners maintain a consistent ratio of speeds throughout the race. Therefore, by the time the winner finishes the race, the other runners will be covering the distance in proportion to their speeds relative to the winner's speed. Here winner covers the length of the tracks if any other conditions are not given.

172. Total degrees in a circle are 360^o. And a clock has been divided into 12 sectors/ gaps (from 1 to 12) so each sector has $360/12$ which is 30^o

173. For every 1 minute the minute hand moves the hour hand moves 0.5^o.

174. The angle between hour hand and a minute hand in a clock is $\theta = \left|\frac{11}{2} min - 30 hours\right|$

Algebraic Identities

175. $(a+b)^2 = a^2 + b^2 + 2ab = (a-b)^2 + 4ab$

176. $(a-b)^2 = a^2 + b^2 - 2ab = (a+b)^2 - 4ab$

177. $(a+b)^3 = a^3 + b^3 + 3(ab)(a+b) = a^3 + 3a^2b + 3ab^2 + b^3$

178. $(a-b)^3 = a^3 - b^3 - 3(ab)(a-b) = a^3 - 3a^2b + 3ab^2 - b^3$

179. $a^2 - b^2 = (a+b)(a-b)$

180. $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

181. $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

182. $a^{2n} - b^{2n} = (a^n - b^n)(a^n + b^n)$

183. If n is odd, then $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots.. + b^{n-1})$

184. If n is odd, then $a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots.. + b^{n-1})$

185. $a^3 + b^3 + c^3 = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$or $(a^3 + b^3 + c^3) - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

     Note: If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

186. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac)$

Quadratic Equations

187. General form: $ax^2 + bx + c = 0$

188. If $\alpha, \beta$are roots $ax^2 + bx + c = 0$ then values of the roots 'α' and 'β' $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

189. If $ax^2 + bx + c = 0$ and '$\alpha\beta$' one the roots of the equation then $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

190. If roots $(\alpha, \beta)$are given, then the quadratic equation will be $(x - \alpha)(x - b) = 0$

191. If sum of the roots$(\alpha + \beta)$ and product of the roots $(\alpha\beta)$ are given, then the quadratic equation will be: $x^2 - (\alpha + \beta) + \alpha\beta = 0$

192. $b^2 - 4ac$is called as the discriminant of a **quadratic** equation.

i) If $b^2 - 4ac$<0 the roots are complex, the roots are imaginary
ii) If $b^2 - 4ac$=0, the roots are real and equal
iii) If $b^2 - 4ac$>0, the roots are real and distinct.
iv) At the same time, the discriminant not being a perfect square, the roots will be conjugate surds of the form $p + \sqrt{q}$ and $p - \sqrt{q}$
v) Discriminant being a perfect square root will be real, rational and unequal.

193. Minimum value of $ax^2 + bx + c = 0$

If a > 0 (an upward parabola): the minimum value = $\frac{4ac-b^2}{4a}$or we also get minimum value on substituting $x = \frac{-b}{2a}$in the quadratic equation

194. If a < 0 (a downward parabola): maximum value of $ax^2 + bx + c = 0 = \frac{4ac-b^2}{4a}$ or at$x = \frac{-b}{2a}$
195. In $ax^2 + bx + c = 0$, if b=c=0, both the roots $\alpha$and βwill be zero
196. The roots are reciprocal to each other if a = c
197. If c = 0, one of the roots will be zero, and the other roots will be $\frac{-b}{a}$

Cubic equations

198. General form = $ax^3 + bx^2 + cx + d = 0$
199. If three roots of $ax^3 + bx^2 + cx + d = 0 (\alpha, \beta, \gamma)$are given then the cubic equation will be $(x-\alpha)(x-\beta)(x-\gamma) = 0$
200. Sum of roots = $\frac{-b}{a} = \alpha + \beta + \gamma$
201. Sum of product of two roots at once is $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
202. Product of roots = $\frac{-d}{a} = \alpha\beta\gamma$
203. If $(\alpha + \beta + \gamma), (\alpha\beta + \beta\gamma + \gamma\alpha)$and $(\alpha\beta\gamma)$are given then the cubic equation will be $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - (\alpha\beta\gamma) = 0$

Polynomials

If $A_n x^n + A_{n-1}x^{n-1} + A_{n-2}x^{n-2} + \ldots\ldots\ldots\ldots + A_1 x + A_0 = 0$, then

204. Sum of the roots: $\frac{-(A_{n-1})}{A_n}$ $(\alpha + \beta + \gamma + \ldots\ldots\ldots + n)$
205. Sum of the product of every two roots =$\frac{-(A_{n-2})}{A_n}$ $(\alpha\beta + \beta\gamma + \alpha\gamma + \ldots\ldots\ldots)$
206. Sum of the product of every three roots = $\frac{-(A_{n-3})}{A_n}$ $(\alpha\beta\gamma + \beta\gamma\alpha + \ldots\ldots\ldots)$
207. Product of the roots =$\frac{(-1)A_0}{A_n}$

Pair of linear Equations: -

208. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is inconsistent with no solution, as they form a pair of parallel lines

209. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is consistent with unique solution, as they intersect at only one point.

210. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. then the pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is consistent with infinite solution, as they are coincident lines.

Important Algebraic Hints: -

x	-3	-2	-1	1	2	3	4	5	6	7	8	9	10
$x + \frac{1}{x}$	-3.33	-2.50	-2	2	2.50	3.33	4.25	5.20	6.167	7.14	8.125	9.11	10.1
$x - \frac{1}{x}$	-2.67	-1.5	0	0	1.50	2.67	3.75	4.80	5.833	6.85	7.875	8.88	9.9

211. If $x + \frac{1}{x} = p$;

$$x^2 + \frac{1}{x^2} = p^2 - 2$$
$$x^3 + \frac{1}{x^3} = p^3 - 3p$$
$$x^4 + \frac{1}{x^4} = (p^2 - 2)^2 - 2$$

212. **If** $x - \frac{1}{x} = p$;

$$x^2 + \frac{1}{x^2} = p^2 + 2$$
$$x^3 + \frac{1}{x^3} = p^3 + 3p$$
$$x^4 + \frac{1}{x^4} = (p^2 + 2)^2 - 2$$

213. If $x + \frac{1}{x} = \sqrt{3}$;

$x^3 + \frac{1}{x^3} = 0$

$$x^6 = -1$$

214. If $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{3}$; $x^3 = -1$

215. If $x + \frac{1}{x} = 1$; then $x^3 = -1$

216. $x + \frac{1}{x} = -1$; then $x^3 = 1$

217. If $x + \frac{1}{x} = 2$then, $x^n - \frac{1}{x^n} = 0$ (By putting x = 1

218. If $x + \frac{1}{x} = 2$then $x^m + \frac{1}{x^n} = 2$ (BY putting x = 1), and $m \neq n$.

219. If $x + \frac{1}{x} = 2$ then $x^m - \frac{1}{x^n} = 0$ (By putting a = 1)

220. If $x + \frac{1}{x} = -2$, then $x^n + \frac{1}{x^n} = 2$If n is even and $x^n + \frac{1}{x^n} = -2$. If n is odd.

221. If $\sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}}$ where x = $n(n+1)$ then $\sqrt{x + \sqrt{x + \sqrt{x + \ldots \infty}}} = (n+1)$

222. If $\sqrt{x-\sqrt{x-\sqrt{x-\ldots\infty}}}$where x = $n(n+1)$ then, $\sqrt{x-\sqrt{x-\sqrt{x-\ldots\ldots\infty}}} = n.$

223. **Binomial theorem:** $(a+b)^n = {}^nc_0a^nb^0 + {}^nC_1a^{n-1}b^1 + {}^nC_2a^{n-2}b^2 + \ldots + nC_{n-1}a^1b^{n-1} + {}^nC_na^0b^n$,

n is a positive number and $nC_r = \frac{n!}{r!(n-r)!}$

224. **Pascal's triangle**

Row 0	---	---	---	---	---	---	1						
Row 1	---	---	---	---	---	1		1					
Row 2	---	---	---	---	1		2		1				
Row 3	---	---	---	1		3		3		1			
Row 4	---	---	1		4		6		4		1		
Row 5	---	1		5		10		10		5		1	
Row 6	1		6		15		20		15		6		1

Special Equations

225. If a and b are factors of k, and if a and b are co-primes, the number of non -negative solutions possible for the linear equation $ax + by = k$is $\frac{k}{ab} + 1$

226. If a and b are factors of k, and if a and b are co-primes, the number of positive solutions possible for the linear equation $ax + by = k$is $\frac{k}{ab} - 1$

227. If one among a or b is a factor of k, and If a and b are co-primes, the number of non - negative solutions possible for the linear equation $ax + by = k$is $\left[\frac{k}{ab}\right] + 1$

228. If one among a or b is a factor of k, and If a and b are co-primes, the number of non - negative solutions possible for the linear equation $ax + by = k$is $\frac{k}{ab}$

Note: - If $\frac{k}{ab}$result is turning out to be in decimals, then take the integers either side of the result.
For eg:- 5.78, then take 6 for non-negative integral solutions and 5 for positive integral solutions.

229. If neither a nor b is a factor of k, in this case, the number of non-negative solutions is equal to possible solutions which is either $\frac{k}{ab}$or $\left[\frac{k}{ab}\right] + 1$ depending on the equation.

230. If $a + b = k$, then $(a)(b)$is maximum when a and b are integers and are close to each other
for eg:- $a + b = 10$, then maximum value of $(a)(b)$= 25, taking a and b, both as 5.
eg:- $a + b = 11$, then maximum value of $(a)(b)$= 30, taking a and b as 5 and 6.

231. If $(a)(b) = k$, then $a + b$is minimum when a and $b$are equal or as close as possible.

232. If A^mB^n = constant → then $(A+B)\frac{A}{m}\frac{B}{n}_{min}$

233. $K_1A + K_2B$ =constant → then $(A^mB^n)\frac{K_1A}{m}\frac{K_2B}{n}_{max}$

234. $A^m B^n$ = constant →then $(K_1 A + K_2 B)\frac{K_1 A}{m}\frac{K_2 B}{n}_{min}$

235. If A + B = constant →then $(A^m B^m)\frac{A}{m}\frac{B}{n}_{max}$

236. $x + \frac{1}{x} \geq 2; x > 0$ →min value of $(1/2)\ |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$at x = 1

237. $x + \frac{1}{x} \leq -2; x < 0$ →max value of $\left(x + \frac{1}{2} = -2\right); x < 0$ ⇒at x = - 1

Indices

238. $a^m a^n = a^{m+n}$

239. $\frac{a^m}{a^n} = a^{m-n}$

240. $(ab)^m = a^m b^m$

241. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

242. $(a^m)^n = a^{mn}$

243. $a^0 = 1$, a ≠ 0

244. $a^1 = 1$

245. $a^{-m} = \frac{1}{a^m}$

246. $a^{m/n} = \sqrt[n]{a^m}$

Surds

247. $\sqrt[n]{ab} = \sqrt[n]{a}\ \ \sqrt[n]{b}$

248. $\sqrt[n]{a}\ \sqrt[m]{b} = \sqrt[nm]{a^m b^n}$

249. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, b ≠ 0

250. $\frac{\sqrt[n]{a}}{\sqrt[m]{b}} = \frac{\sqrt[nm]{a^m}}{\sqrt[nm]{b^n}} = \sqrt[nm]{\frac{a^m}{b^n}}$, b ≠ 0.

251. $\left(\sqrt[n]{a^m}\right)^p = \sqrt[n]{a^{mp}}$
252. $\left(\sqrt[n]{a}\right)^n$ = a
253. $\left(\sqrt[n]{a^m}\right) = \sqrt[np]{a^{mp}}$
254. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
255. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
256. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$
257. $\boldsymbol{\frac{1}{\sqrt[n]{a}} = \frac{\sqrt[n]{a^{n-1}}}{a}}$**, a ≠ 0.**
258. $\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$
259. $\frac{1}{\sqrt{a} \pm \sqrt{b}} = \frac{\sqrt{a} \mp \sqrt{b}}{a - b}$
260. A quadratic surd, let's say "$\sqrt{a}$", cannot be equal to the sum or difference of a quadratic surd and a rational number ut's say another quadratic surd $\sqrt{h}$ and 'c' be a rational number $\sqrt{a} \neq \sqrt{b} + c$ (or) $\sqrt{a} \neq \sqrt{b}$ – c
261. If a +$\sqrt{b} = c + \sqrt{d}$ or a - $\sqrt{b}$ = c - $\sqrt{d}$, then a = c and b = d
262. If a $\pm \sqrt{b}$ = 0 then a = b = 0
263. If a + $\sqrt{b}$ = c + $\sqrt{d}$, then a - $\sqrt{b} = c - \sqrt{d}$ and its vice-versa
264. If $\sqrt{a + \sqrt{b}} = \sqrt{c} + \sqrt{d}$, then $\sqrt{a\sqrt{b}} = \sqrt{c} - \sqrt{d}$ and its vice versa

Logarithms

If $a^x = b$

Then $log_a b = x$where $a \neq 1, a > 0, b > 0$

Eg: $2^7 = 128 \Rightarrow log_2 1\,28 = 7$

265. A logarithm with base 10 is called a common logarithm
266. $log_a 1 = 0$
267. $log_a a = 1 \;\; a \neq 1, a > 0$
268. $log_a a^x = x$
269. $a^{log\, a^x} = x$
270. $log_a(mn) = log_a m + log_a n m, n > 0;\; a > 0;\; a \neq 1$
271. $log_a\left(\frac{m}{n}\right) = log_a m - log_a n$
272. $log_a m^n = n\, log_a m \qquad m > 0, a > 0, a \neq 1$
273. $log_a\left(\frac{1}{m}\right) = -log_a m \qquad m > 0, a > 0, a \neq 1$
274. $log_a b = \frac{1}{log_b a} = \frac{log_c b}{log_c a} \qquad \forall a, b, c > 0$and $a \neq 1, b \neq 1, c \neq 1$
275. If $log_a b = x$;
    a) $log_{1/a} 1\,/b = x$
    b) $log_{1/a} b = -x$

c) $log_a 1/b = -x$

276. $log_{a^m}(b) = \frac{1}{m} log_a b$

277. $log_{a^m}(b^n) = \frac{n}{m} log_a b$

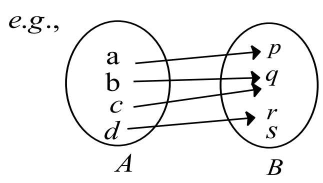
278. If 0<a<1, $log_a x$is a decreasing function

279. If a>1, $log_a x$ is an increasing function

Functions and Graphs

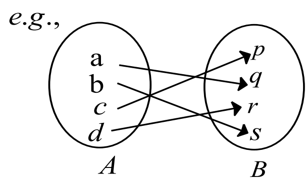
280. **Surjective and Injective Functions**

Many inputs produce one output



Many-one

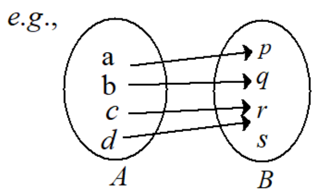
Only one input produce any one output i.e., no two inputs produce same output



One-one

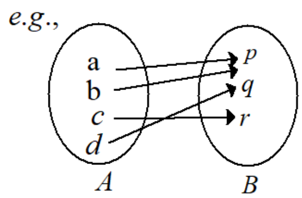
Into

There are at least one element of B which is not the output (or image) of any element of A.



Onto (or surjective)

Every element of B is necessarily an output (or image) of an element of A.



281. **One to One (or Injective) Functions**

A function / is said to be one-to-one if it does not take the same values at two distinct points in its domain. For example, $f(x) = x^3$ is one to one whereas $f(x) = x^2$ is not, as $f(1) = 1$ and $f(-1) = 1$

282. Function which are not one to one or called many one function

283. One many function does not exist.

284. **Onto (or Surjective Function)**

If a function $f : A \rightarrow B$ is such that each clement in B is the f-image of at least one element in A, then we say that is f is function of A 'onto' B. Equivalently a function f is an onto function if codomain of f = Range of f.

In order to show that a function $f: A \rightarrow B$ is onto we start with any $y \in B$ and try to find $x \in A$ such that $f(x) = y$.

285. If there exists at least one element in B which is not the f-image of any element of A, then f is called an into function.

286. **Bijective Function:** If a function f is both one-to-one and onto then f is said to be a bijective function. e.g., an identity function is a bijective function.

287. If a set A has m elements and set B has n elements, then the number of functions possible from A to B is n^m.

288. **Number of Surjective Functions (Onto Functions):** If a set A has m elements and set B has n elements, then the number of onto functions from A to B = $n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - \ldots\ldots - {}^nC_{n-1}(1)^m$; (m > n)

289. **Number of Injective Functions (One to One):** If set A has n elements and set B has m elements, m≥n, then the number of injective functions or one to one function is given by m!/(m-n)!.

290. **Number of Bijective functions:** If there is a bijection between two sets, A and B, then both sets will have the same number of elements. If n(A) = n(B) = m, then the number of bijective functions = m!.

Progressions

291. $a_1, a_2, a_3, a_4, a_5, \ldots, a_n$ are 'n' terms said to be in arithmetic progression when there is a common difference between every two consecutive terms.

292. Let 'a' be the first term, and 'd' be the common difference, then $a,\ a + d,\ a + 2d,\ a + 3d,\ a + 4d, \ldots, a + (n - 1)d$ are 'n' terms which are in AP

293. n^{th} term of AP

$$[a_n] = a + (n-1)d$$

294. Sum of 'n' terms in AP $= \frac{n}{2}[firstterm + lastterm] = S_n$ or $S_n = \frac{n}{2}[2a + (n-1)d]$

295. Whenever, there are three terms which are in AP. The middle term is the average of the three terms or the middle term is the average of the other two terms.

Let's say a, b, c are in AP $b = \frac{a+c}{2}$or 2b = a + c

Note: whenever in the problems, the sum of three terms in AP is 's' is given, take three as $a - d,\ a,\ a + d$ instead of $a,\ a + d,\ a + 2d$

296. The sum of first ‘n’ natural numbers is $\frac{n(n+1)}{2}$, The average of first ‘n’ natural numbers is $\frac{(n+1)}{2}$

297. The sum of first ‘n’ odd numbers is n^2, the average of first ‘n’ odd numbers is n.

298. The sum of first ‘n’ even numbers is $(n+1)n$, the average of first ‘n’ even number is n+1.

299. The sum of square of first ‘n’ natural numbers is $\frac{n(n+1)(2n+1)}{6}$

300. The sum of cubes of first ‘n’ natural numbers is $\left(\frac{n(n+1)}{2}\right)^2$

301. A sequence a_1, a_2, a_3, …… an are said to be in GP. $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots . \frac{a_n}{a_{n-1}} = r$

‘r’ here is common ratio

E.g. 2, 6, 18, 54,…. [This series is in GP], as the common ratio is ‘3’

GP = a, ar, ar^2, ar^3, ……. [representation of series in GP]

302. nth term in GP

$T_n = a.r^{(n-1)}$

When ‘n’ is number of terms / nth term ‘r’ is common ratio and ‘a’ is first term

sum of first ‘n’ terms in GP

$s_n = \frac{a.(r^n-1)}{(r-1)}$, if r>1 or $S_n = \frac{lr-a}{r-1}$, where l is last term

$S_n = \frac{a(1-r^n)}{(1-r)}$, if r < 1

when r = 1, formula will not work

303. Sum of infinite terms in a GP

$S_\infty = \frac{a}{(1-r)}$: $|r| < 1$

304. Geometric mean $= \sqrt[n]{productofterms}$

305. If three terms are in GP, the middle one is the Geometric mean of the other two terms suppose a, b, c are in then = b^2 = ac or b = $\sqrt{ac}$

306. A sequence a_1, a_2, a_3, …… a_n are said to be in HP (harmonic progression) when sequence is formed by reciprocals of a_1, a_2, a_3, …… a_n, and a_1, a_2, a_3, …… a_n are in A.P

The terms in H.P ; $\frac{1}{a} + \frac{1}{a+d} + \frac{1}{a+2d} + \ldots$

307. $n^{th}\, term$ in H.P $= \frac{1}{a+(n-1)d}$

308. Harmonic Mean of $[a_1, a_2, a_3 \ldots\ldots a_n] = \frac{4}{\frac{1}{a_1}+\frac{1}{a_2}+\ldots+\frac{1}{a_n}}$

309. AM $\geq$GM $\geq$ HM

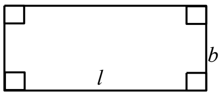
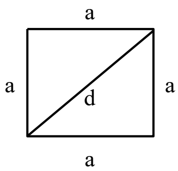
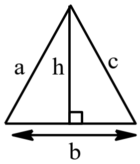
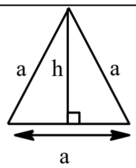
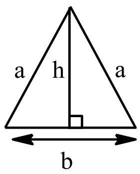
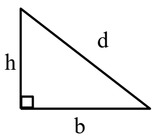
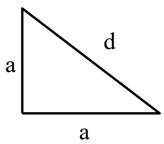
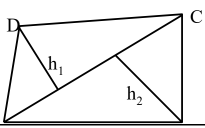
310. If AM = GM = HM only when all the observation are real and equal

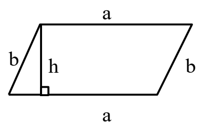
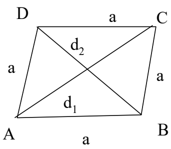
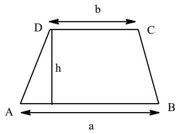
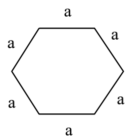
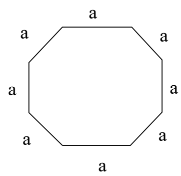
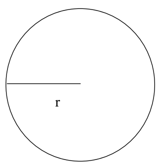
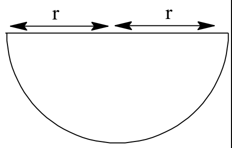
311. If a, b, c, are there observations, then $\frac{a+b+c}{3} \geq \sqrt[3]{abc} \geq \frac{3}{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}$

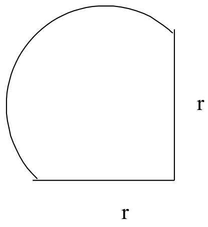
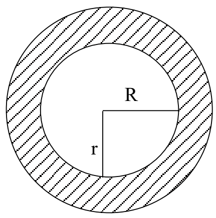
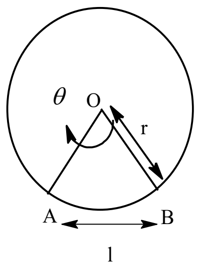
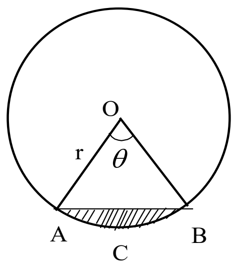
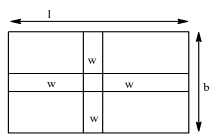
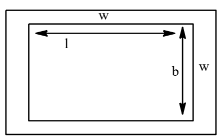
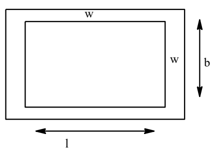
Series and Sequence

312. $t_1 + t_2 + t_3 + t_4 + \ldots + t_n = \sum t_n$
313. $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$
314. $2 + 4 + 6 + \ldots + 2n$ = n (n + 1)
315. $1 + 3 + 5 + \ldots + (2n - 1)$ = n^2
316. $k + (k + 1) + (k + 2) + \ldots + (k + n - 1) = \frac{n(2k+n-1)}{2}$
317. $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$
318. $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$
319. $1^2 + 3^2 + 5^2 + \ldots + (2n - 1)^2 = \frac{n(4n^2-1)}{3}$
320. $1^3 + 3^3 + 5^3 + \ldots + (2n - 1)^3 = n^2 (2n^2 - 1)$
321. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} + \cdots = 2$
322. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} + \ldots = 1$
323. $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{(n-1)!} + \cdots = e$
324. Power Series in x: $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n + \ldots$
325. Power Series in $(x - x0)$: $\sum_{n=0}^{\infty} a_n (x - x_0)^n = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + \ldots a_n(x - x_0)^n + \ldots$

Mensuration

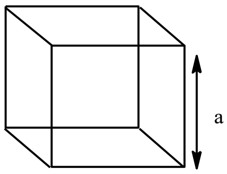
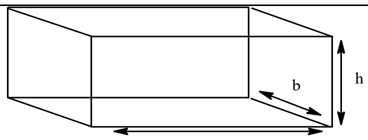
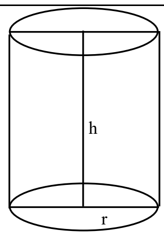
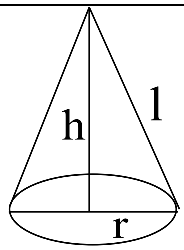
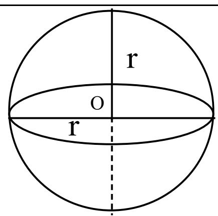
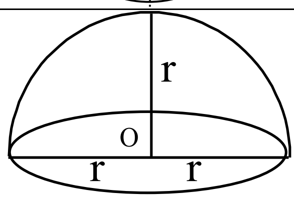
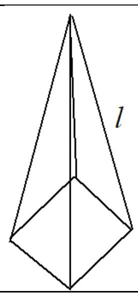
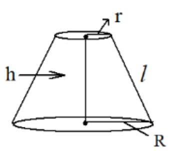
Name	Figure	Nomenclature	Area	Perimeter
Rectangle		l → length b → breadth	l × b = lb	2l + 2b = 2(l + b)
Square		a → side d → diagonal d = a√2	a × a = a^2 $\frac{d^2}{2}$	a + a + a + a = 4a
Triangle		a, b and c are three sides of triangle an s is the semi perimeter, where s = $\frac{a+b+c}{2}$ b is the base and h is altitude of triangle	$\frac{1}{2} \times b \times h$ $\sqrt{s(s-a)(s-b)s-c)}$	a + b + c = 2s
Equilateral Triangle		a → sides h → Height or altitude h =$\frac{\sqrt{3}}{2}a$	$\frac{1}{2} \times a \times h$ $\frac{\sqrt{3}}{4}a^2$	3a
Isosceles triangle		a → equal sides b → base h = $\frac{\sqrt{4a^2-b^2}}{2}$ h → height or altitude	$\frac{1}{2} \times b \times h$ $\frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$	2a + b
Right angled triangle		b → base h → altitude/height d → hypotenuse d= $\sqrt{b^2 + h^2}$	$\frac{1}{2} \times b \times h$	b + h + d
Isosceles Right angled triangle		a → equal sides d → hypotenuse d = a√2	$\frac{1}{2}a^2$	2a + d
Quadrilateral		AC is the diagonal and h_1, h_2 are the altitudes on AC	$\frac{1}{2} \times AC \times (h_1 + h_2)$	AB+BC+CD+AD

		from the vertices D and B respectively		
Parallelogram		a and b are sides adjacent to each other h→ distance between the parallel sides	a × h	2(a + b)
Rhombus		a → each equal side of Rhombus d_1 and d_2 are the diagonals d_1 → BD d_2 → AC	$\frac{1}{2} \times d_1 \times d_2$	4a
Trapezium		a and b are parallel sides to each other and h is the perpendicular distance between parallel sides	$\left(\frac{a+b}{2}\right) \times h$	AB+BC+CD+AD
Regular hexagon		a → each of the equal sides	$\frac{3\sqrt{3}}{2} a^2$	6a
Regular octagon		a → each of equal sides	$2a^2(1+\sqrt{2})$	8a
Circle		r → radius of the circle π= $\frac{22}{7} = 3.1416$ (approx.)	πr^2	2πr (called as circumference)
Semicircle		r → radius of the circle	$\frac{1}{2}\pi r^2$	πr + 2r

Quadrant		r → radius	$\frac{1}{4}\pi r^2$	$\frac{1}{2}\pi r + 2r$
Ring of circular path (shaded region)		R → Outer radius r → inner radius	$\pi(R^2 - r^2)$	Outer→ 2πR inner→2πr
Sector of a circle		O → center of the circle r → radius l → length of the arc θ→ angle of the sector l = $2\pi r\left(\frac{\theta}{360^o}\right)$	$\pi r^2\left(\frac{\theta}{360^o}\right)$ $\frac{1}{2}r \times 1$	1 + 2r
Segment of a circle		θ→ angle of the sector r → radius AB → chord ACB → arc of the circle	Area of segment ACB (Minor segment) = $r^2\left(\frac{\pi\theta}{360^o} - \frac{\sin\theta}{2}\right)$	$\left[\frac{\theta}{360^o} \times 2\pi r + 2r\sin\left(\frac{\theta}{2}\right)\right]$
Pathways running across the middle of a rectangle		l → length b → breadth w → width of the path(road)	(l + b – w)w	$2(l+b) - 4w + 4w =$ $2(l+b)$
Outer pathways		l → length b → breadth w → widthness of the path	(l + b + 2w)2w	Outer→ $2(l+b)$ inner→$2(l+b+4w)$
Inner path		l → length b → breadth w → width of the path	(l + b – 2w)2w	Outer→ $2(l+b)$ Inner→$2(l+b-4w)$

326. **Volume of a Prism (Cube, Cuboid, Cylinder, etc) = Area of base × Height**

327. **Volume of a Pyramid (Cone, Right Cone, etc)** $=\frac{1}{3}$ **Area of base × Height**

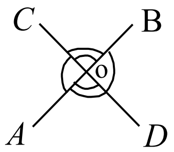
Cube		• Edge of cube = length = breadth = height = a • Volume of a cube = $(edge)^3 = a^3$ • Total surface area = 6×$(edge)^2 = 6a^2$ • Diagonal of a cube = $\sqrt{3} \times edge = \sqrt{3}a$
Cuboid		• V = l × b × h • Surface area = 2(lb + bh + hl) • Diagonal = $\sqrt{l^2 + b^2 + h^2}$
Right Circular Cylinder		• Volume = $\pi r^2 h$ • Curved Surface Area = $2\pi rh$ • Total surface Area = $2\pi rh + 2\pi r^2 = 2\pi r(r + h)$
Right Circular Cone		• Volume = $\frac{1}{3}\pi r^2 h$ • Curved Surface Area = πrl • Total Surface Area = $\pi rl + \pi r^2$ = pπ(l+ r)
Sphere		• Volume = $\frac{4}{3}\pi r^3$ • Surface Area = $4\pi r^2$
Hemisphere		• Volume = $\frac{2}{3}\pi r^3$ • Curved Surface Area = $2\pi r^2$ • Total surface Area = $2\pi r^2 + \pi r^2 = 3\pi r^2$
Right Quadrilateral Pyramid		• Volume = Area of base × Height • Curved Surface Area = $\frac{1}{2} \times l \times (Perimeter of base)$ • Total Surface Area = $\frac{1}{2} \times l \times (Perimeter of base) + area of the base$
Frustum Cone		• Volume = $\frac{\pi}{3}h(r^2 + Rr + R^2)$ • Curved Surface Area = $\pi(r + R)l$ • Total Surface Area = $\pi(r + R)l + \pi(R^2 + r^2)$

Geometry

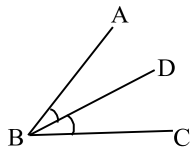
Lines and angles

328. Complementary angles $[\theta_1, \theta_2] \Longrightarrow \theta_1 + \theta_2 = 90°$
329. Supplementary angles $[\theta_1,\ \theta_2] \Longrightarrow \theta_1 + \theta_2 = 180°$
330. **Vertically opposite angles: -**

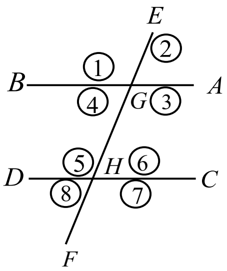
$\angle AOD = \angle BOC$
$\angle BOD = \angle AOC$



331. **Angular bisector: -** BC is the angular bisector of $\angle ABC$ if the line divides the angle into two equal parts $\angle ABD = \angle DBC = \frac{1}{2}(\angle ABC)$



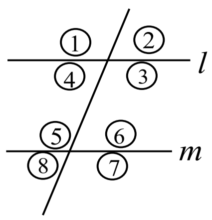
332. **Corresponding angles: -** When two non-parallel lines are intersected by a transversal, then they form four pairs of corresponding angles



(A) $\angle AGE, \angle CHG \Rightarrow (\angle 2, \angle 5)$ (B) $\angle AGH, \angle CHF \Rightarrow (\angle 3, \angle 7)$
(C) $\angle EGB, \angle GHD \Rightarrow (\angle 1, \angle 5)$ (D) $\angle BGH, \angle DHF \Rightarrow (\angle 4, \angle 8)$

333. **Exterior angles** $\Rightarrow \angle 1, \angle 2, \angle 7, \angle 8$
334. **Interior angles** $\Rightarrow \angle 3, \angle 4, \angle 5, \angle 6$
335. **Pair of alternate angles** $\Rightarrow \angle 3, \angle 5$and $\angle 6, \angle 4$
336. When two parallel lines are intersected by a transversal, the pair of corresponding angles so formed are congruent

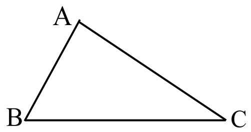
$$\left.\begin{array}{ll}(a) & \angle 1 = \angle 5 \\ (b) & \angle 2 = \angle 6 \\ (c) & \angle 4 = \angle 8 \\ (d) & \angle 3 = \angle 7\end{array}\right\} \text{Corresponding Angles}$$



(e) $\angle 1 = \angle 5 = \angle 3 = \angle 7$
(f) $\angle 2 = \angle 6 = \angle 4 = \angle 8$
(g) $\angle 3 = \angle 5 \& \angle 4 = \angle 6$ are pair of Alternate Interior angles
(h) $\angle 1 = \angle 7 \& \angle 2 = \angle 8$are pair of Alternate Exterior angles

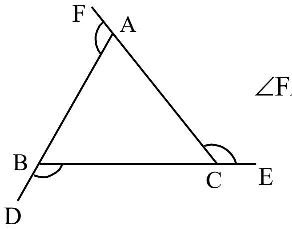
Triangles

337. Sum of any two sides is always grater then the third side.
338. Difference of any two sides is always less than the third side.
339. In ΔABC, BC be the largest side then



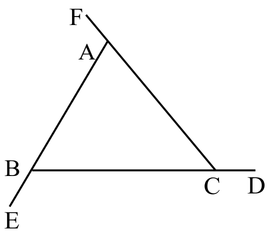
(a) If $BC^2 = AB^2 + AC^2$,then $\Delta ABC$is right – angled triangle
(b) If $BC^2 < AB^2 + AC^2$, then ΔABC is acute – angled triangle.
(c) If $BC^2 > AB^2 + AC^2$, then ΔABC is obtuse angle triangle.

340. Sum of all three interior angles is always 180°
341. Sum of all three exterior angles is always 360°



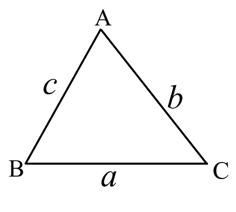
$\angle FAB + \angle DBC + \angle ACE = 360°$

342. In a triangle ABC the measure of exterior angle equals the sum of the measures of the interior opposite angles



(a) $\angle BAF = \angle ABC + \angle ACB$
(b) $\angle CBF = \angle ACB + \angle BAC$
(c) $\angle ACD = \angle CAB + \angle ABC$

343. **Sin rule:** In a ΔleABC, let a, b, c be the three sides opposite to the angles A, B, C respectively, then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



344. **Cosine rule:** $Cos\ A = \frac{b^2+c^2-a^2}{2bc}$; $Cos\ B = \frac{c^2+a^2-b^2}{2ac}$; $Cos\quad C = \frac{a^2+b^2-c^2}{2ab}$

Areas of Triangles:

345. $\Delta = \sqrt{S(S-a)(S-b)(S-c)}$; where S = $\frac{a+b+c}{2}$Heron's formula
346. $\Delta = \frac{1}{2}\times$base × height

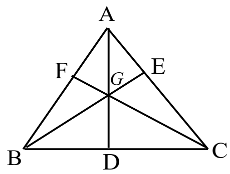
347. $\Delta = \frac{1}{2} \sin A\,(bc) = \frac{1}{2} \sin B\,(ac) = \frac{1}{2} \sin c\,(ab)$

348. Δ = S.r; where 'r' is the inradius of the triangle and

349. $\Delta = \frac{abc}{4R}$; 'R'is the circumradius of the triangle

Centres of a Triangles:

350. **Centroid**: The centroid is a point where three medians of a triangle intersect. A median of a triangle is a line segment joining a vertex to the midpoint of the opposite side.



→ AD, BE & CF are medians

→ 'G' is the centroid of ΔABC

351. $\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$; Centroid divides each median in the ratio 2 : 1

352. Here, Area of 6 triangles hence formed have equal areas, when 'G' is the centroid of the ΔABC.

353. A median divides the area of Δle into two equal halves.

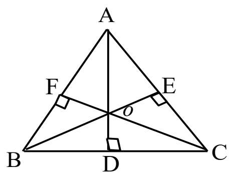
354. **Orthocentre :** -The orthocenter of a triangle is the point where the three altitudes (perpendicular lines from each vertex to the opposite side) of the triangle intersect.

Position in Different Types of Triangles:

i) In an acute triangle, the orthocenter lies inside the triangle.

ii) In an obtuse triangle, the orthocenter lies outside the triangle.

iii) In a right triangle, the orthocenter coincides with one of the vertices (the vertex opposite the hypotenuse).



→AD, BE and ∠CF are the altitudes

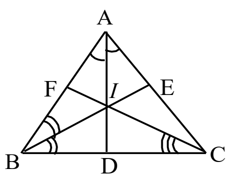
→ O is the orthocentre

355. ∠BOC = 180° − ∠A

356. ∠AOC = 180° − ∠B

357. ∠AOB = 180° − ∠C

358. **Incentre** :- The incentre of a triangle is the point where the three angle bisectors of the triangle intersect. An angle bisector is a line segment that divides an angle into two equal angles.



→ AD, BE & CF are angel bisectors of ∠A, ∠B & ∠C respectively.

→ I is the incentre.

$\angle$ BAD = $\angle$CAD = $\frac{1}{2}\angle A$

$\angle$CBE = $\angle$ABE = $\frac{1}{2}\angle$B

$\angle$ACF = $\angle$BCF = $\frac{1}{2}\angle$C

$\angle$BIC = 90° + $\frac{1}{2}\angle$A

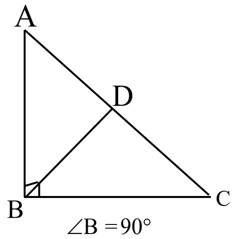
$\angle$AIC = 90° + $\frac{1}{2}\angle B$

$\angle$AIB = 90° + $\frac{1}{2}\angle C$

359. **Circumcenter**:- The circumcenter of a triangle is the point where the perpendicular bisectors of the sides of the triangle intersect. A perpendicular bisector is a line that is perpendicular to a side of the triangle and passes through its midpoint. The circumcenter may lie inside, outside, or on the triangle, depending on the type of triangle (acute, obtuse, or right, respectively).

→ AD, BE & CF are perpendicular bisectors of $\angle A, \angle B \& \angle C$ respectively.
→ O is the circumcenter.
$\angle$BOC = 2 $\angle$A; $\angle$ AOC = 2 $\angle$B; $\angle$A O B = 2 $\angle$C

360. **Pythagoras theorem:-**
The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides. i.e., $(AC)^2 = (AB)^2 + (BC)^2$
The numbers which satisfy this relation, are called Pythagorean triplets.
e. g,(3, 4, 5),
(5, 12, 13),
(7, 24,25),
(8, 15,17),
(9, 40, 41),
(11, 60, 61),
(12, 35, 37),
(16, 63, 65),



$\angle B = 90°$

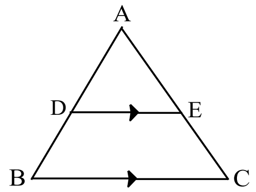
$\overrightarrow{AC} \rightarrow$ Hypotenuse

AD = CD = BD

(D is the mid-point of AC)

361. All the multiples (or submultiples) of Pythagorean triplets also satisfy the relation. e.g., (6, 8,10), (15,36, 39), (1.5, 2, 2.5) etc

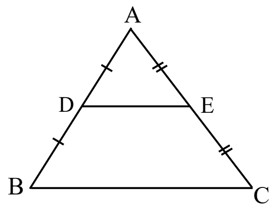
362. **Basic proportionality theorem (BPT) or Thales theorem**
Any line parallel to one side of a triangle divides the other two sides proportionally. So if DE is drawn parallel to BC, it would divide sides AB and AC proportionally i.e.,



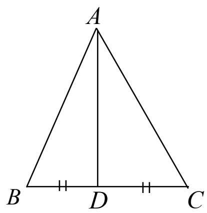
$$\frac{AD}{DB} = \frac{AE}{EC} \text{ or } \frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{AD}{DE} = \frac{AB}{BC} = \frac{AE}{DE} = \frac{AC}{BC}$$

363. **Mid – point theorem:** If the mid-points of t wo adjacent sides of a triangle are joined by a line segment, this segment is parallel to the third side. i.e., if AD = BD and AE = CE, then DE II BC



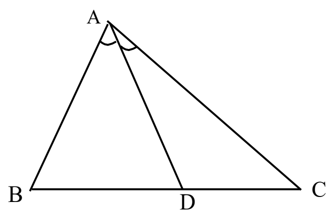
364. **Apollonius theorem: -** In a triangle, the sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side. i.e., $AB^2 + AC^2 = 2(AD^2 + BD^2)$



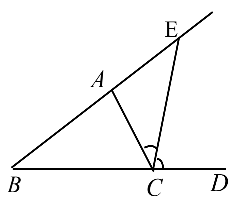
$BD = CD$

AD is the median

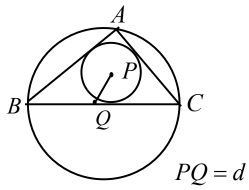
365. **Interior angle bisector theorem: -** In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. i.e., $\frac{BD}{CD} = \frac{AB}{AC}$and $BD \times AC - CD \times AB = AD^2$



366. **Exterior angle bisector theorem: -** In a triangle the angle bisector of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of the remaining two sides i.e., $\frac{BE}{AE} = \frac{BC}{AC}$

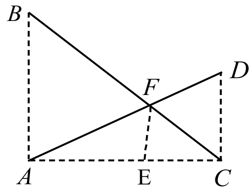


367. **Euler's Theorem for a triangle: -** Let ΔABC have circumradius R and inradius r. Let d be the distance between the circumcentre and the incentre. Then we have $d^2 = R(R - 2r)$

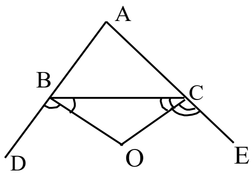


$PQ = d$

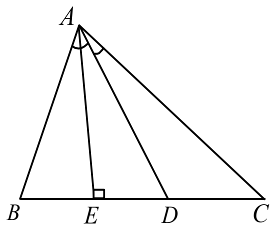
368. **Crossed Ladder Theorem: -** Let the two-line segments BC and AD intersect at a point F, such that the point E lies on AC and $AB||CD||EF$. Then, we have $\frac{1}{AB}+\frac{1}{CD}=\frac{1}{EF}$



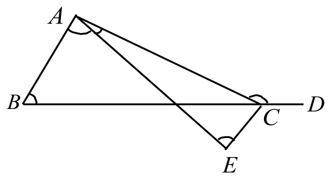
369. In a ΔABC, if sides AB and AC are produced to D and E respectively and the bisectors of ∠DBC and ∠ECB intersect at O, then ∠BOC = 90°-$\frac{1}{2}$∠A



370. In a ΔABC, if AD is the angle bisector of ∠BAC and AE ⊥BC, then $\angle DAE = \frac{1}{2}(\angle ABC - \angle ACB)$

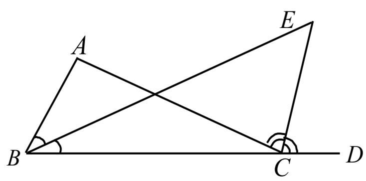


371. In a ΔABC, if BC is produced to D and AE is the angle bisector of ∠A, then



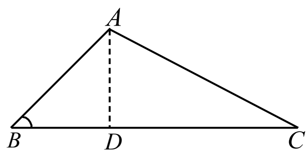
$\angle ABC + \angle ACD = 2\angle AEC.$

372. In a ΔABC, if side BC is produced to D and bisectors of $\angle ABC$ and $\angle ACD$ meet at E, then



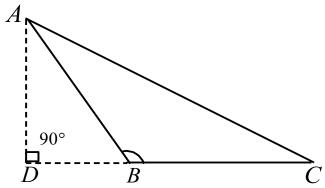
$\angle BEC = \frac{1}{2}\angle BAC$

373. In an acute angle ΔABC, AD is a perpendicular dropped on the opposite side of ∠A, then



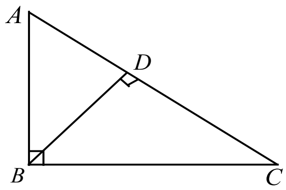
$$AC^2 = AB^2 + BC^2 - 2.BD.BC(\angle B < 90°)$$

374. In an obtuse angle ΔABC, AD is perpendicular dropped on BC, BC is produced to D to meet AD, then



$$AC^2 = AB^2 + BC^2 + 2BD.BC(\angle B > 90°)$$

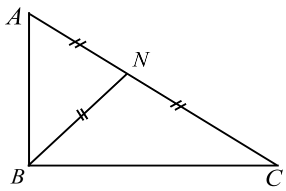
375. In a right angle ΔABC, ∠B = 90° and AC is hypotenuse. The perpendicular BD is dropped on hypotenuse AC from right angle vertex B, then



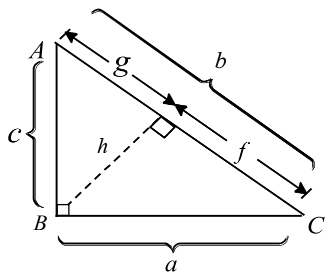
(i) $BD = \frac{AB \times BC}{AC}$ (ii) $AD = \frac{AB^2}{AC}$

(iii) $CD = \frac{BC^2}{AC}$ (iv) $\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$

376. In a right-angled triangle, the median to the hypotenuse = $\frac{1}{2}$ ×hypotenuse i.e., $BN = \frac{AC}{2}$

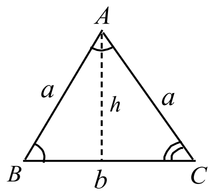


Right Triangle:-



377. $b^2 = a^2 + c^2$
378. $a^2 = b \times f$
379. $c^2 = b \times g$
380. $h^2 = f \times g$
381. $ac = bh$
382. Circumradius ⇒ $R = \frac{b}{2}$
383. Inradius ⇒ $R = \frac{a+c-b}{2} = \frac{ac}{a+b+c}$
384. Area ⇒ $\frac{1}{2} \times a \times c$

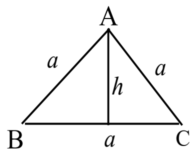
Isosceles Triangle:



In an isosceles triangle ABC, with sides a, b, c, let b = c, then

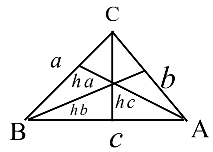
385. $h^2 = a^2 - \frac{b^2}{4}$
386. Perimeter = a + b + c
387. Area = $\frac{1}{2} \times b \times h = \frac{1}{2} \times sin\, A \times a^2$
388. Height = $\sqrt{a^2 - \frac{b^2}{4}}$

Equilateral Triangle: -



389. Height = $\frac{\sqrt{3}}{2}(a)$
390. Area = $\frac{1}{2} \times h \times a = \frac{\sqrt{3}}{4}(a^2)$
391. Perimeter = 3 (a)
392. Circumradius ⇒ R = $\frac{2}{3}(h) = \frac{a\sqrt{3}}{3}$
393. Inradius ⇒ $r = \frac{1}{3}(h) = \frac{R}{2} = \frac{a\sqrt{3}}{6}$

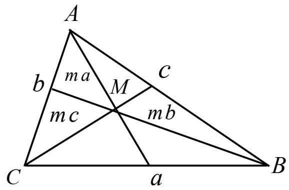
Scalene Triangle: -



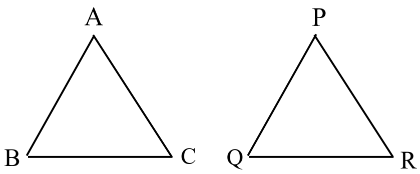
394. Property of scalene triangle

 $(a + b) > c;$
 $(b + c) > a;$
 $(a + c) > b$
 $| a - b| < c;\ |b - c| < a;\ |a - c| < b$

395. Sem perimeter = S = $\frac{a+b+c}{2}$
396. Height 1 ⇒ $h_a =$ b sin C = c sin B = $\frac{2}{b}\sqrt{S(s-a)(s-b)(s-c)}$
397. Height 2 ⇒ h_b = a sin C = c sin A = $\frac{2}{b}\sqrt{S(s-a)(s-b)(s-c)}$
398. Height 3 ⇒ h_c = a sin B = b sin A = $\frac{2}{c}\sqrt{S(s-a)(s-b)(s-c)}$



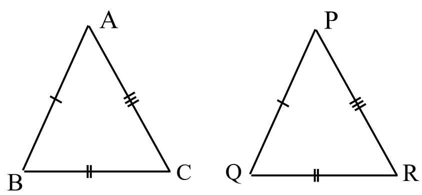
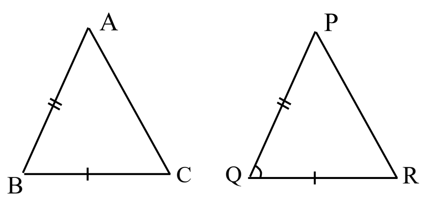
399. Median 1 ⇒ $m_a = \frac{b^2+c^2}{2} - \frac{a^2}{4}$
400. Median 2 ⇒ $m_b = \frac{a^2+c^2}{b} - \frac{b^2}{4}$
401. Median 3 ⇒ $m_c = \frac{a^2+b^2}{2} - \frac{c^2}{4}$
402. $AM = \frac{2}{3}(m_a)$
403. $BM = \frac{2}{3}(m_b)$
404. $CM = \frac{2}{3}(m_c)$
405. Area = $\sqrt{S(s-a)(s-b)(s-c)}$ where S = $\frac{a+b+c}{2}$ [herons Formula]
406. **Congruency of triangles:** Two triangles are said to be congruent if they are equal in all respects. i.e.,

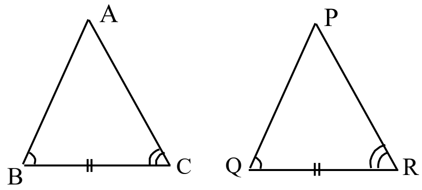
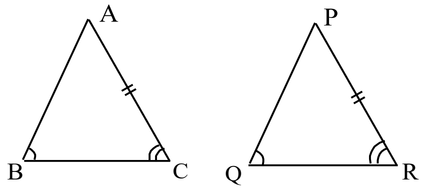
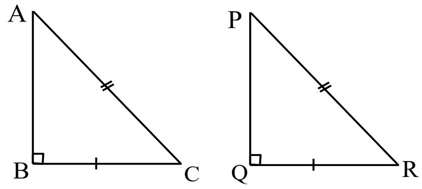


1. Each of the three sides of one triangle must be equal to the three respectively sides of the other.
2. Each of the three angles of the one triangle must be equal to three respective angles of the other.

i.e., $\left.\begin{array}{l} AB = PQ \\ AC = PR \\ BC = QR \end{array}\right\}$ and $\left.\begin{array}{l} \angle A = \angle P \\ \angle B = \angle Q \\ \angle C = \angle R \end{array}\right\}$

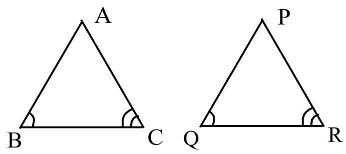
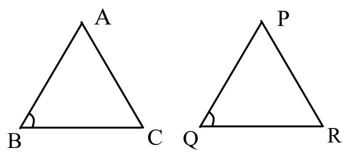
407.

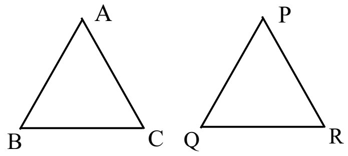
Test	Property	Diagram
s – s - s	(side – side-side) If the three sides of one triangle are equal to the corresponding three sides of the other triangle, then the two triangles are congruent. AB ≅ PQ, AC ≅ PR, BC ≅ QR $\therefore\ \Delta ABC \cong \Delta PQR$	
s – A - s	(Side – Angle -Side) If two sides and the angle included between them are congruent to the corresponding sides and the angle included between them, of the other triangle, then the two triangles are congruent. $AB \cong PQ, \angle ABC \cong \angle PQR, BC \cong QR$ $\therefore\ \Delta ABC \cong \Delta PQR$	

A – S - A	(Angle – Side – Angle) If two angles and the included side of a triangle are congruent to the corresponding angles and the included side of the other triangle, then the two triangles are congruent. $\angle ABC \cong \angle PQR, BC \cong QR, \angle ACB = \angle PRQ$ $\therefore \Delta ABC \cong \Delta PQR$	
A – A – S	(Angle- Angle – Side) If two angles and a side other than the included side of a one triangle are congruent to the corresponding angles and a corresponding side other than the included side of the other triangle, then the two triangles are congruent. $\angle ABC \cong \angle PQR, \angle ACB \cong \angle PRQ$ and $AC \cong PR$ (or AB ≅PQ)	
R – H – S	(Right angle – Hypotenuse – Side) If the hypotenuse and one side of the right angled triangle are congruent triangle, then the two given triangles are congruent. $AC \cong PR, \angle B = \angle Q$ and BC ≅ QR $\therefore \Delta ABC \cong \Delta PQR$	

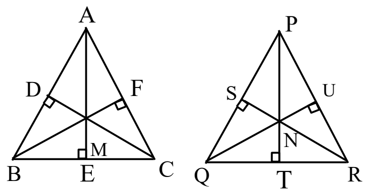
408. **Similarity of triangles:** Two triangles are said to be similar if the corresponding angles are congruent and their corresponding sides are in proportion. The symbol for similarity is '~'. If $\Delta ABC \sim \Delta PQR$, then $\angle ABC \cong \angle PQR, \angle BCA \cong \angle QRP, \angle BAC \cong \angle QPR$

409. **Tests for Similarity:**

Test	Property/Definition	Diagram
A – A	Angle – Angle If t he two angles of one triangle are congruent to the corresponding two angles of the other triangle, then the two triangles are said to be similar. $\angle ABC \cong \angle PQR$ $\angle ACB \cong \angle PRQ$ $\therefore \Delta ABC \sim \Delta PQR$	
S – A – S	Side – Angle – Side If the two sides of one triangle are proportional to the corresponding two sides of the other triangle and the angle included by them are congruent, then the two triangles are similar. i.e., $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle ABC = \angle PQR$ $\therefore \Delta ABC \sim \Delta PQR$.	 $\frac{AB}{PQ} = \frac{BC}{QR} = K$ (K is any constant)

S - S – S	Side – Side – Side If the three sides of one triangle are proportional to the corresponding three sides of the tother triangle, then the two triangles are similar. i.e., $$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}$$ $\therefore \Delta ABC \sim \Delta PQR$	 $\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}=K$

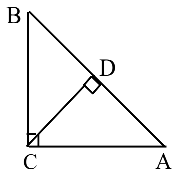
410. **Properties of Similar Triangles :-** If the two triangles are similar, then



Ratio of sides = Ratio of heights (altitudes) = Ratio of medians = Ratio of angles bisectors = Ratio of inradii = Ratio of circumradii

411. If the two triangles are similar, then ratio of areas = ratio of squares of corresponding sides, i.e., if $\Delta ABC \sim \Delta PQR$, then $\frac{A(\Delta ABC)}{A(\Delta PQR)}=\frac{(AB)^2}{(PQ)^2}=\frac{(BC)^2}{(QR)^2}=\frac{(AC)^2}{(PR)^2}$

412. In a right-angled triangle, the triangles on each side of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to the original triangle and to each other too. i.e., $\Delta BCA \sim \Delta BDC \sim \Delta CDA$.



413. Total no. of triangles that can be formed using the other two sides ⇒ 2 [minimum side length] – 1

414. If the perimeter of the triangle is odd, then the number of distinct triangles formed are $\left[\frac{(P+3)^3}{48}\right]$; where P is the perimeter and [ ] is the nearest integer function.

For example : - Perimeter = 27 cm.

$$\left[\frac{(27+3)^2}{48}\right]=[18.75]=19$$

Note : - [3.8] = 4 $|[4.5]=4|[7.1]=7$

415. If the perimeter of the triangle is even, the number of distinct triangles formed are $\left[\frac{P^2}{48}\right]$; where ‘P’ is the perimeter and [ ] is the nearest integer function.

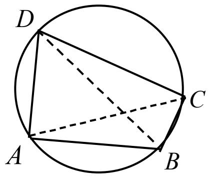
416. If the perimeter of a Δ is P the number of distinct scalene triangles to can be formed are $\left[\frac{(P-3)^2}{48}\right]$; if the perimeter is odd and $\left[\frac{(P-6)^2}{48}\right]$; if the perimeter is even.

417. If the perimeter of a Δ is P, the number of Isosceles Δ les can be formed are ⇒ Total, no. of triangles – [scalene triangle + Equilateral Triangles]

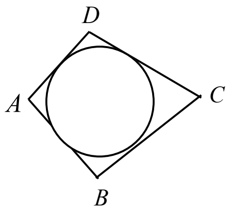
418. If the perimeter is constant, the maximum area of the triangle is possible when an equilateral triangle is formed.

Circles

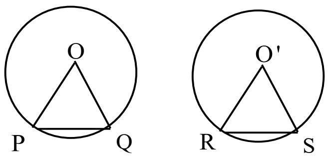
419. A circle is a set of points on a plane which lie at a fixed distance from a fixed point.
420. **Cyclic Quadrilateral: -** A quadrilateral whose all the four vertices (A, B, C, D) lie on the circle. For a cyclic quadrilateral ABCD, we must have $(AB \times CD) + (BC \times AD) = (AC \times BD)$



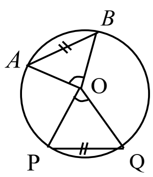
421. **Tangential Quadrilateral:** A tangential quadrilateral or circumscribed quadrilateral is a convex quadrilateral whose sides are all tangent to a single circle within the quadrilateral.
For a tangential quadrilateral, we must have AB + CD = BC + AD



422. Two arcs of a circle (or of congruent circles) are congruent if their degree measures are equal.
423. There is one and only one circle passes through three non – collinear points.
424. In a circle (or in congruent circles) equal chords are made by equal arcs.$\{OP = OQ\} = \{O'R = O'S\}$ and PQ = RS
∴ PQ = RS

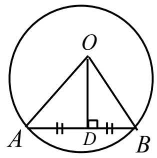


425. Equal arcs (or chords) subtend equal angles at the centre
∵ PQ = AB
∴ $\angle POQ = \angle AOB$



426. The perpendicular from the centre of a circle to a chord bisects the chord i.e., if $OD \perp AB$

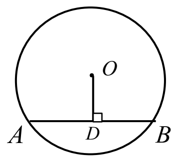
$$\therefore\ AB = 2AD = 2BD$$



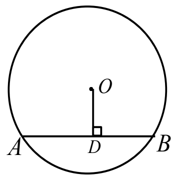
427. The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

∵ AD = DB

∴ $OD \perp AB$



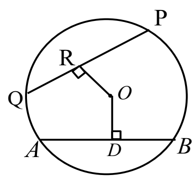
428. Perpendicular bisector of a chord passes through the centre. i.e., if $OD \perp AB$ and AD = DB∴ O is the centre of the circle.



429. Equal chords of a circle (or of congruent circles) are equidistant from the centre.

∵ AB = PQ

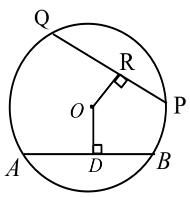
∴ OD = OR



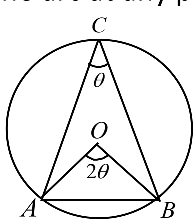
430. Chords which are equidistant from the centre in a circle (or in congruent, circles) are equal.

∵ OD = OR

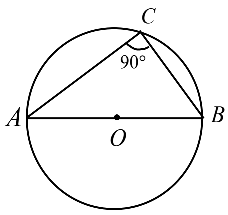
∴ AB = PQ



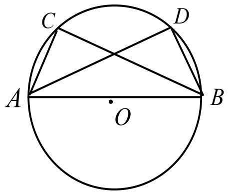
431. The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle.



432. Angle in a semicircle is a right angle.



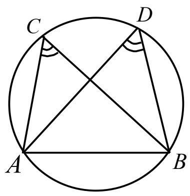
433. Angles in the same segment of a circle are equal. i.e., $\angle \mathrm{ACB} = \angle ADB$



434. If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle.

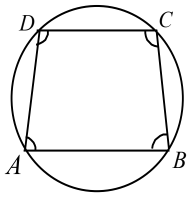
$$\angle \mathrm{ACB} = \angle ADB$$

∴ Points A, C, D, B are concyclic i.e., lie on the circle.



435. The sum of pair of opposite angles off a cyclic quadrilateral is 180°.

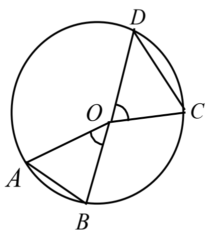
$\angle DAB + \angle BCD = 180°$and $\angle ABC + \angle CDA = 180°$ (Inverse of this theorem is also true)



436. Equal chords (or equal arcs) of a circle (or congruent circles) subtended equal angles at the centres.

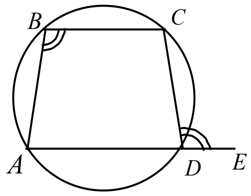
AB = CD

∴ $\angle AOB = \angle COD$ (Inverse of this theorem is also true)

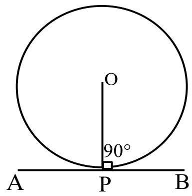


437. If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle.

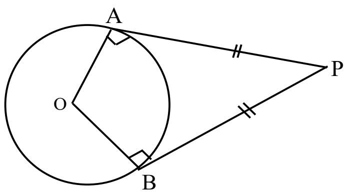
$m\angle CDE = m\angle ABC$



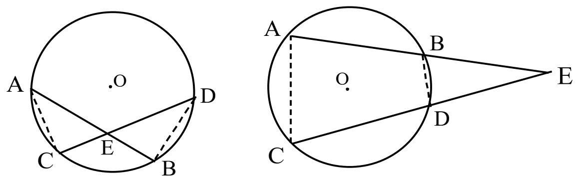
438. A tangent at any point of a circle is perpendicular to the radius through the point of contact. (Inverse of this theorem is also true)



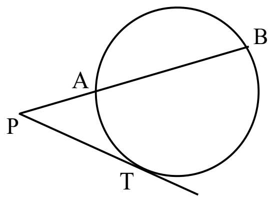
439. The lengths of two tangents drawn from an external point to a circle are equal i.e., AP = BP



440. If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E), then AE × BE = CE ×DE

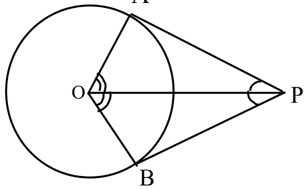


441. If PB be a secant which intersects the circle at A and B and PT be a tangent at T, then PA. PB = $(PT)^2$



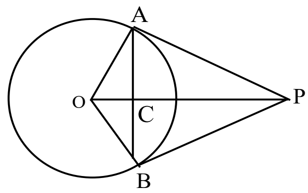
442. From an external point from which the tangents are drawn to the circle with centre O, then

(a) they subtend equal angles at the centre.
(b) they are equally inclined to the line segment joining the centre of that point.
$\angle AOP = \angle BOP$ and $\angle APO = \angle BPO$



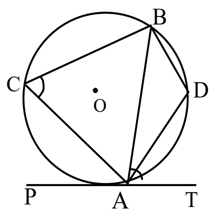
443. If P is an external point from which the tangents to the circle with centre O touch it at A and B, then OP is the perpendicular bisector of AB.

$OP \perp AB$ and AC = BC



444. **Alternate segment theorem:** If from the point of contact off a tangent, a chord is drawn, then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram.

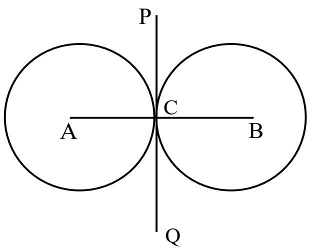
$\angle BAT = \angle BCA$ and $\angle BAP = \angle BDA$

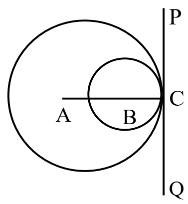


445. The point of contact of two tangents lies on the straight line joining the two centres.

(a) When two circles touch externally, then the distance between their centres is equal to sum off their radii. i.e., AB = AC + BC

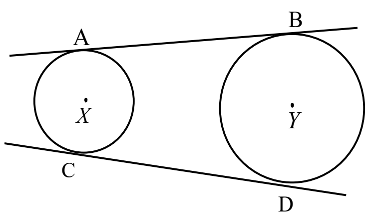
(b) When two circles touch internally, the distance between their centres is equal to the difference between their radii. i.e., $AB = AC - BC$



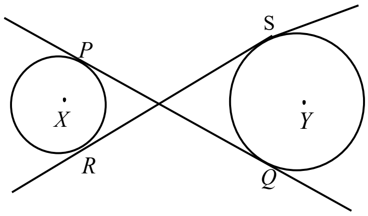


446. For the two circles with centre X and Y and radii r_1 and r_2. AB and CD are two Direct Common Tangents (DCT), then the length of DC=

$\sqrt{(\text{distancebetweencentres})^2 - (r_1 - r_2)^2}$

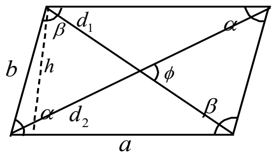


447. For the two circles with centre X and Y and radii $r_1$and r_2. PQ and RS are two transverse common tangent, then length of TCT = $\sqrt{(\text{distance between centres})^2 - (r_1 + r_2)^2}$



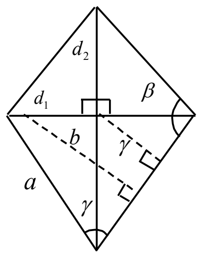
Quadrilaterals:-

Parallelogram :-



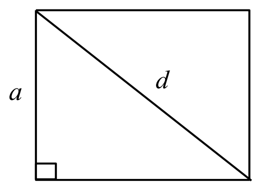
448. $\alpha + \beta = 180°$
449. h = b sin α = b
450. Perimeter = 2 (a + b)
451. Area = ah = ab sin d = $\frac{1}{2}d_1 d_2\, sin\, \phi$
452. $d_1^2 + d_2^2 = 2(a^2 + b^2)$

Rhombus :-



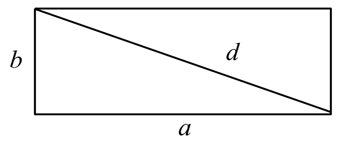
453. $\alpha + \beta = 180$
454. $h = a\, sin\, \alpha = \frac{d_1 d_2}{2a}$
455. $r = \frac{h}{2} = \frac{d_1 d_2}{4a} = \frac{a\, sin\, \alpha}{2}$
456. Perimeter = 4a
457. Area = $\frac{1}{2}d_1 d_2 = ah = a^2\, sin\, \alpha$
458. $d_1^2 + d_2^2 = 4a^2$

Square:-

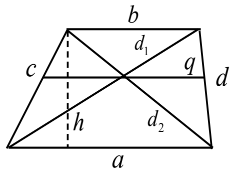


459. $d = a\sqrt{2}$
460. $R = \frac{d}{2} = \frac{a\sqrt{2}}{2}$
461. $r = \frac{a}{2}$
462. Perimeter = 4a
463. Area = a^2

Rectangle: -

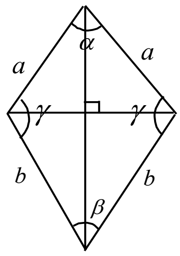


464. $d = \sqrt{a^2 + b^2}$
465. $R = \frac{d}{2}$
466. Perimeter = $2(a + b)$
467. Area = ab



468. q= $\frac{a+b}{2}$
469. Area = $\frac{a+b}{2}.h = qh$
470. $d_1^2 + d_2^2 = c^2 + d^2 + 2ab$

Kite: -



471. $\alpha + \beta + 2\gamma = 360°$
472. Perimeter = $2[a + b]$
473. Area = $\frac{d_1 d_2}{2}$
474. $r = \frac{Area}{a+b}$

Regular polygons : - Let 'n' be the number of sides of a regular polygon work 'α' is each interior angle of the polygon and with each side 'a'.

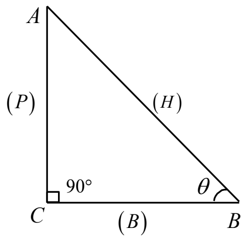
475. $\alpha = \left[\frac{n-2}{n}\right] 180°$
476. R = $\frac{a}{2\,sin\left[\frac{180}{n}\right]}$
477. r = $\frac{a}{2+tan\left(\frac{180}{n}\right)} = \sqrt{R^2 - \frac{a^2}{4}}$
478. $Perimeter = na$
479. Area = $r.\left[\frac{na}{2}\right] = \frac{na}{2}\sqrt{R^2 - \frac{a^2}{4}} = \frac{nR^2}{2}.sin\frac{2\pi}{n}$
480. Exterior angle = $\left[\frac{360}{n}\right]^{\circ}$
481. Number of diagonals = $\frac{n(n-3)}{2}$
482. Sum of all interior angles = $[n - 2]180°$

Trigonometry

483. **Trigonometrical Ratios**

$sin\,\theta = \frac{P}{H}, cos\,\theta = \frac{B}{H}, tan\,\theta = \frac{P}{B}$

$cos\,ec\theta = \frac{H}{P}, sec\,\theta = \frac{H}{B}, cot\,\theta = \frac{B}{P}$



$sin^2\,\theta + cos^2\,\theta = 1 \Rightarrow sec^2\,\theta - tan^2\,\theta = 1 \Rightarrow cos\,ec^2\theta - cot^2\,\theta = 1$

484. **Some of the trigonometric Values**

Angle	$\theta = 0$	θ = 30°	θ = 45°	θ = 60°	θ = 90°	θ = 180°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞

485. $cos\,ec\theta = \frac{1}{sin\,\theta}$
486. $sec\,\theta = \frac{1}{cos\,\theta}$
487. $cot\,\theta = \frac{1}{tan\,\theta}$
488. $tan\,\theta = \frac{sin\,\theta}{cos\,\theta}$
489. $cot\,\theta = \frac{cos\,\theta}{sin\,\theta}$
490. $tan\,\theta = \frac{1}{cot\,\theta}$
491. $cot\,\theta = \frac{1}{tan\,\theta}$

Range of Trigonometric Ratios

492. $-1 \leq sin\,\theta \leq 1 \Rightarrow |sin\,\theta| \leq 1$
493. $-1 \leq cos\,\theta \leq 1 \Rightarrow |cos\,\theta| \leq 1$
494. $cos\,ec\theta \leq -1$and $cos\,ec\theta \geq 1 \Rightarrow |cos\,ec\theta| \geq 1$
495. $sec\,\theta \leq -1$ and $sec\,\theta \geq 1 \Rightarrow |sec\,\theta| \geq 1$

496. $-\infty < tan\,\theta < \infty$ i.e., $tan\,\theta$ may assume any value.

497. **Increasing and Decreasing Functions of T-Ratios**

	1st quadrant	**2nd quadrant**	**3rd quadrant**	**4th quadrant**
sin θ	increases from 0 to 1	decreases from 1 to 0	decreases from 0 to − 1	increases from − 1 to 0
cos θ	decreases from 1 to 0	decreases from 0 to − 1	increases from − 1 to 0	increases from 0 to 1
tan θ	increases from 0 to ∞	increases from −∞ to 0	increases from 0 to ∞	increases from −∞ to 0

498. **Trigonometrical Ratios of Negative and Associated Angles**

Angle	**- θ**	**(90 - θ)**	**(90 + θ)**	**(180 - θ)**	**(180 + θ)**	**(360 - θ)**	**(360 + θ)**
sin θ	-sin θ	cos θ	cos θ	sin θ	-sin θ	-sin θ	sin θ
cos θ	cos θ	sin θ	-sin θ	-cos θ	-cos θ	cos θ	cos θ
tan θ	-tan θ	cot θ	-cot θ	-tan θ	tan θ	-tan θ	tan θ

Sum, Difference and Product Formulae

499. $sin\,(A + B) = sin\,A\,cos\,B + cos\,A\,sin\,B$
500. $sin\,(A - B) = sin\,A\,cos\,B - cos\,A\,sin\,B$
501. $cos\,(A + B) = cos\,A\,cos\,B - sin\,A\,sin\,B$
502. $cos\,(A - B) = cos\,A\,cos\,B + sin\,A\,sin\,B$
503. $tan\,(A + B) = \frac{tan\,A + tan\,B}{1 - tan\,A\,tan\,B}$
504. $tan\,(A - B) = \frac{tan\,A - tan\,B}{1 + tan\,A\,tan\,B}$
505. $sin(A + B)\,sin(A - B) = sin^2 A - sin^2 B = cos^2 B - cos^2 A$
506. $cos(A + B)\,cos(A - B) = cos^2 A - sin^2 B = cos^2 B - sin^2 A$
507. $2\,sin\,A\,cos\,B = sin\,(A + B) + sin\,(A - B)$
508. $2\,cos\,A\,sin\,B = sin\,(A + B) - sin\,(A - B)$
509. $2\,cos\,A\,cos\,B = cos\,(A + B) + cos\,(A - B)$
510. $2\,sin\,A\,sin\,B = cos\,(A - B) - cos\,(A + B)$
511. $cot(A + B) = \frac{cot\,A\,cot\,B - 1}{cot\,A + cot\,B}$
512. $cot(A - B) = \frac{cot\,A\,cot\,B + 1}{cot\,B - cot\,A}$
513. $sin\,C + sin\,D = 2\,sin\frac{C+D}{2}\,cos\frac{C-D}{2}$
514. $sin\,C - sin\,D = 2\,cos\frac{C+D}{2}\,sin\frac{C-D}{2}$
515. $cos\,C + cos\,D = 2\,cos\frac{C+D}{2}\,cos\frac{C-D}{2}$
516. $cos\,C - cos\,D = 2\,sin\frac{C+D}{2}\,sin\frac{D-C}{2}$

Trigonometrical Ratios of Multiple and Sub-multiple Angles

517. $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1+\tan^2 A}$

518. $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \frac{1-\tan^2 A}{1+\tan^2 A}$

519. $\tan 2A = \frac{2 \tan A}{1-\tan^2 A}$

520. $\sin 3A = 3 \sin A - 4 \sin^3 A$

521. $\cos 3A = 4 \cos^3 A - 3 \cos A$

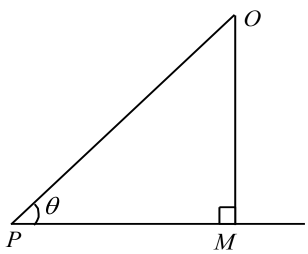
522. $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3\tan^2 A}$

523. $\sin A = 2 \sin\left(\frac{A}{2}\right) \cos\left(\frac{A}{2}\right) = \frac{2\tan\left(\frac{A}{2}\right)}{1+\tan^2\left(\frac{A}{2}\right)}$

524. $\cos A = \cos^2\left(\frac{A}{2}\right) - \sin^2\left(\frac{A}{2}\right)$etc.

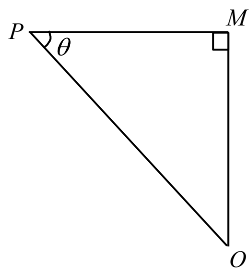
525. $\tan A = \frac{2\tan\left(\frac{A}{2}\right)}{1-\tan^2\left(\frac{A}{2}\right)}$

526. **Angle of Elevation:** $\angle OPM$is called as the angle of elevation.



Suppose a person P is looking an object O which is at a higher level than P, then the angle θ ($\angle OPM$)is the angle of elevation for that person.

527. **Angle of Depression:** ($\angle OPM$)is called as angle of depression.



Coordinate Geometry

528. Distance Between Two Points d = AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
529. Midpoint of a Line Segment$x_0 = x\frac{x_1+x_2}{2}, y_0 = \frac{y_1+y_2}{2}, \lambda = 1.$
530. Centroid (Intersection of Medians) of a triangle

   $x_0 = \frac{x_1+x_2+x_3}{3}, y_0 = \frac{y_1+y_2+y_3}{3},$

   Where A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) are vertices of the triangle ABC.
531. Incenter (Intersection of Angle Bisectors) of a triangle

   $x_0 = \frac{ax_1+bx_2+cx_3}{a+b+c}, y_0 = \frac{ay_1+by_2+cy_3}{a+b+c}$, Where a = BC, b = CA, c = AB.
532. Circumcenter (Intersection of the Side Perpendicular Bisectors) of a triangle

$$x_0 = \frac{\begin{vmatrix} x_1^2+y_1^2 & y_1 & 1 \\ x_2^2+y_2^2 & y_2 & 1 \\ x_3^3+y_3^3 & y_3 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, y_0 = \frac{\begin{vmatrix} x_1 & x_1^2+y_1^2 & 1 \\ x_2 & x_2^2+y_2^2 & 1 \\ x_3 & x_3^3+y_3^3 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$

533. Orthocenter (Intersection of Altitudes) of a triangle

$$x_0 = \frac{\begin{vmatrix} y_1 & x_2x_3+y_1^2 & 1 \\ y_2 & x_3x_1+y_2^2 & 1 \\ y_3 & x_1x_2+y_3^2 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, y_0 = \frac{\begin{vmatrix} x_1^2+y_2y_3 & x_1 & 1 \\ x_2^2+y_3y_1 & x_2 & 1 \\ x_3^2+y_1y_2 & x_3 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}$$

534. Area of a triangle S = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = (\pm)\frac{1}{2}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}$

   = $(1/2)\ |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$
535. Area of a quadrilateral S = ($\frac{1}{2}[(x_1 - x_2)(y_1 + y_2) + (x_2 - x_3)(y_2 + y_3) +$
   $(x_3 - x_4)(y_3 + y_4) + (x_4 - x_1)(y_4 + y_1y)]$
536. Slope of a Line(m) = tan $\alpha = \frac{y_2 - y_1}{x_2 - x_1}$
537. Equation of a line that passes through two points: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$ (or) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$
538. Intercept Form: $\frac{x}{a} + \frac{y}{b}$ = 1
539. Normal Form: x$\cos\beta + y\sin\beta - p = 0$
540. Vertical Line X = a
541. Horizontal Line Y = b
542. **Parallel Lines:** Two lines y = $\mathrm{m_1x + b_1}$ and y = $\mathrm{m_2x + b_2}$ are parallel if $\mathrm{m_1 = m_2}$.

   Two lines $\mathrm{A_1x + B_1y + C_1 = 0}$ and $\mathrm{A_2x + B_2y + C_2 = 0}$ are parallel if $\frac{\mathrm{A_1}}{\mathrm{A_2}} = \frac{\mathrm{B_1}}{\mathrm{B_2}}$.

543. **Perpendicular Lines:**

Two lines y = $m_1x + b_1$ and y = $m_2x + b_2$ are perpendicular if $m_2 = -\frac{1}{m_1}$ or, equivalently, $m_1m_2 = -1$.

Two lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2$ = 0 are perpendicular if

$$A_1A_2 + B_1B_2 = 0.$$

544. **Distance of a Point From a Given Line or Length of Perpendicular From the Point (x_1, y_1) to the Straight Line ax + by + c = 0**

Let the length of perpendicular be p, then $p = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Let the two parallel lines be $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$, then the perpendicular distance between the lines is $\left|\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right|$

545. **Intersection of Two Lines**

If two lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2$ = 0 intersect, the intersection point has coordinates

$x_0 = \frac{-C_1B_2 + C_2B_1}{A_1B_2 - A_2B_1}, y_0 = \frac{-A_1C_2 + A_2C_1}{A_1B_2 - A_2B_1}$.

Section Formula

546. If R(x, y) divides the line segment joined by the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio m : n then $x = \frac{mx_2+nx_1}{m+n}, y = \frac{my_2+ny_1}{m+n}$

547. If R (x, y) divides the line segment joined by the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ externally in the ratio m: n, then $x = \frac{mx_2-nx_1}{m-n}, y = \frac{my_2-ny_1}{m-n}$

548. **Angle Between two Lines:** $tan \quad \theta = \pm\left(\frac{m_2-m_1}{1+m_1m_2}\right)$; where m_1 and m_2 are slopes of two lines.

549. **Two Point Form:** The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is $y - y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x - x_1)$

550. Let $a_1x + b_1y + c_1 = 0$ .....(i)

$a_2x + b_2y + c_2 = 0$ ....(ii)

$a_3x + b_3y + c_3 = 0$ ....(iii)

Be three concurrent lines then

$a_3(b_1c_2 - b_2c_1) + b_3(c_1a_2 - c_2a_1) + c_3(a_1b_2 - a_2b_1) = 0$

$$\Rightarrow \quad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

551. **Equations of Straight Lines Passing Through (x_1, y_1) making angle θ with the given line y = mx + c**

$$y - y_1 = \frac{m \pm tan\,\theta}{1 \mp m\,tan\,\theta}(x - x_1)$$

Permutations and Combinations

552. The number of permutations that can be formed by taking r things at a time from a set of n distinct things is denoted by nP_r, for every $1 \leq r \leq$n.

$$nP_r = n(n-1)(n-2)(n-3)....(n-r+1) = \frac{n!}{(n-r)!}$$

553. n! = 1.2.3.4…(n – 1).n
554. (n – r)! = 1.2.3.4….(n – r – 2) (n – r – 1) (n – r)
555. 0! = 1 = 1!

Permutations of n Different Things

556. Number of permutations of n different things taken all at a time = $nP_n = n!$
557. Number of permutations of n different things taken r at a time =$nP_n = \frac{n!}{(n-r)!}$
558. Number of permutations of n different things taken at most r at a time =
$$nP_1 + {}^nP_2 + {}^nP_3 + \ldots + {}^nP_r$$
559. Number of permutations of n different things taken at least r at a time =
$$nP_r + {}^nP_{r+1} + {}^nP_{r+2} + \ldots + {}^nP_n$$
560. Number of permutations of n different things taken r at a time, when one particular thing always occurs = $r.(n - rP_{r-1})$
561. Number of permutations of n different things taken r at a time, when one particular thing never occurs = $n - 1P_r$
562. Number of permutations of n different things taken r at a time, when k particular things never occur = $(rP_k)(n - kP_{r-k})$
563. Number of permutations of n different things taken r at a time, when k particular things never occur = $n - kP_r$
564. Number of permutations of n different things, taken all at a time, when m specified things always come together = $m!\,(n - m + 1)!$
565. Number of permutations of n different things, taken all at a time, when m specified things never come together = $n! - [m!\ (n - m + 1)!]$

Permutations of n Things Not All Different

566. Number of permutations of n things taken all at a time when p of them are all alike and the rest are all different = $\frac{n!}{p!}$
567. Number of permutations of n things taken all a time when p things are alike of one kind, q things are alike of other kind, r things are alike of another kind and rest all, if any, are different from p, q, r, then the number of permutations of n things = $\frac{n!}{p!q!r!}$
568. Number of permutations of n things taken all at a time when p_1 things are alike of one kind, $p_2$things are alike of second kind, p_3 things are alike of third kind, ……. p_r things are alike of r^{th} kind, and rest all, if any, are different from $p_1, p_2, p_3 \ldots\ldots, p_r$, then the number of permutations of n things = $\frac{n!}{p_1!p_2!p_3!.....p_r!}$

Permutations Where Repetitions are Allowed

569. Number of permutations of n different things taken exactly r at a time, when each may be repeated any number of times in each arrangement = n^r

570. Number of permutations of n different things taken at most r at a time, when each may be repeated any number of times in each arrangement $= n + n^2 + n^3 + \ldots\ldots + n^r = \frac{n(n^r-1)}{n-1}$

571. Number of permutations of n different things taken at least r at a time, when each may be repeated any number of times in each arrangement

$$= n^r + n^{r+1} + n^{r+2} + \ldots\ldots + n^n = \frac{n^r(n^{(n-r+1)}-1)}{n-1}$$

Circular Permutations

572. Number of circular permutations of n different things taken all at a time= (n- 1)!

573. Number of circular permutations of n different things taken all at a time in one direction(either clock-wise or anti-clock-wise) == $\frac{(n-1)!}{2}$

574. Number of circular permutations of n different things taken r at a time= $\frac{nP_r}{r}$

575. Number of circular permutations of n different things taken r at a time in one direction (either clock-wise or anti-clock-wise) == $\frac{nP_r}{2r}$

Combinations

576. The number of combinations of n dissimilar things taken r at a time is denoted by nC_r or $C(n, r)$ or $\binom{n}{r}$; where$nC_r = \frac{n!}{r!(n-r)!}$; Whereas C denotes the combination and$r \leq n$

577. $nC_r = \frac{nP_r}{r!}$

578. $nC_o = ^n C_n = 1$

579. $nC_r = ^n C_{n-r} (0 \leq r \leq n)$

580. $nC_x = ^n C_y \Leftrightarrow x + y = n$ or $x = y; (x, y \in W)$

581. $nC_{r-1} + ^n C_r = ^{n+1} C_r$

582. If n is even, the greatest value of $nC_r = ^n C_m$ where $m = n/2$

583. If n is odd, the greatest value of $nC_r = ^n C_m$where $m = \frac{(n-1)}{2}$or m = $\frac{(n+1)}{2}$

584. $rC_r + ^{r+1}C_r + ^{r+2}C_r + \ldots\ldots + ^n C_r = ^{n+1} C_{r+1}; r \leq n$

585. $nC_0 + ^n C_1 + ^n C_2 + ^n C_3 + \ldots. + ^n C_n = 2^n$

586. $nC_1 + ^n C_2 + ^n C_3 + \ldots. + ^n C_n = 2^n - 1$

587. $nC_0 + ^n C_2 + ^n C_4 + \ldots. = 2^{n-1}$

588. $nC_1 + ^n C_3 + ^n C_5 + \ldots. = 2^{n-1}$

Combination of n Different Things

589. Number of combinations of n different things taken r at a time, when x particular things always occur = $n - xC_{r-x}$

590. Number of combinations of n different things taken r at a time, when x particular things never occur =$n - xC_r$

591. Number of combinations of n different things taken r at a time, when x particular things always occur and y particular things never occur = $n - x - yC_{r-x}$

592. Number of combinations of n different things taken r at a time, when x particular things are not together in any selection = $n - xC_{r-x}$

593. Number of ways of selections of zero or more things from a group of n distinct things

= $nC_0+^nC_1+^nC_2+^nC_3+\ldots..+^nC_n = 2^n$

594. Number of ways of selections of one or more things from a group of n distinct things

= $nC_1+^nC_2+^nC_3+\ldots..+^nC_n = 2^n - 1(\because^n C_0 = 1)$

Combination of n Identical Things

595. Number of ways of selections of r things from a group of n identical things = 1.
596. Number of ways of selections of at most r things from a group of n identical things = $r\ +1$
597. Number of ways of selections of at least r things from a group of n identical things = $(n - r) + 1$
598. Number of ways of selections of at least one thing from a group of n identical things = n
599. Number of ways of selections of zero or more things from a group of n identical things = $n\ +1$

Combination of n Things Not All Different

Number of ways of selection of some or all the things from a group of distinct subgroups:

600. Number of ways of selection of zero or more things out of $(\, p\ +\ q\ +\ r+\ldots)$ things, of which p are alike of one kind, q are alike of second kind, r are alike of third kind, and so on = $[(p\ +\ 1\ )\ (q\ +1\ )\ (r\ +\ 1\ )\ldots]$
601. Number of ways of selection of at least one thing out of $(p\ +\ q\ +\ r+\ldots)$ things, of which p are alike of one kind, q are alike of second kind, r are alike of third kind, and so on = $[(p\ +\ 1)\ (q\ +\ 1)\ (r\ +\ 1)\ \ldots..]\ -\ 1$
602. Number of ways of selection of zero or more things from p identical things of one kind, q identical things of second kind, r identical things of third kind, and from remaining n things which are all different $= [(p+1)(q+1)(r+1)2^n]$
603. Number of ways of selection of at least one thing from p identical things of one kind, q identical things of second kind, r identical things of third kind, and from remaining n things which are all different$= [\{(p+1)(q+1)(r+1)2^n\} - 1]$
604. The number of ways of choosing k objects, from p objects of one kind, q objects of second kind, and so on, is the coefficient of x^k in the expansion$(1+x+x^2+\ldots+x^p)(1+x+x^2+\ldots\ldots+x^q)\ldots..$
605. The number of ways of choosing k objects from p objects of one kind, q objects of second kind, and so on, such that one object of each kind must be included is the coefficient of x^k in the expansion

$(x+x^2+\ldots..+x^p)(x+x^2+\ldots\ldots+x^q)\ldots..$

606. The number of ways in which r things can be selected from a group of n things of which p are identical is

$\sum_0^r n - pC_r$;if r $\leq\ p$

607. The number of ways in which r things can be selected from a group of n things of which p are identical is

$\sum_{r=p}^r n - pC_r$; if > p

Combination of Contiguous Things

608. I. Number of selections of k consecutive things out of n things in a row = $(n - k + 1)$

II. Number of selections of k things out of n things in a row, such that no two of the selected objects are next to each other = $n - k + 1C_k$

609. **Circular Combination**

Number of selections of k consecutive things out of n things in a circle =$\begin{cases} n, when k < n \\ 1, when k = n \end{cases}$

Distribution/Division of Distinct Things Among Individuals/Groups

610. Number of ways in which (m + n) different things can be divided into two unequal groups containing m and n things, if the order of the group is not important

$={}^{m+n} C_m \times {}^{n} C_n = \frac{(m+n)!}{m!n!}$

611. Number of ways in which (m + n) different things can be divided into two unequal groups containing m and n things, if the order of the group is important$= \frac{(m+n)!}{m!n!} \times 2!$

612. The number of ways in which (m + n + p) different things can be divided into three different groups containing m n, and p things respectively, if the order of the group is not important

$={}^{m+n+p} C_m \times {}^{n+p} C_n \times {}^{p} C_p = \frac{(m+n+p)!}{m!n!p!}$

613. The number of ways in which (m + n + p) different things can be divided into three different groups containing m n, and p things respectively, if the order of the group is important $=\frac{(m+n+p)!}{m!n!p!} \times 3!$

614. The number of ways in which m different items can be divided equally into m groups, each containing n objects when the order of the group is not important$= \left[\frac{(mn)!}{(n!)^m}\right] \frac{1}{m!}$

615. The number of ways in which mn different items can be divided equally into m groups, each containing n objects when the order of the groups is important$= \left[\frac{(mn)!}{(n!)^m}\right]$

616. The number of ways of dividing $2n$ different things into three groups of n each when the order of groups is not important $= \frac{(2n)!}{(n!)^2} \frac{1}{2!}$ the division by 2! indicates that since 2 groups have equal number of things, so the two groups are indistinguishable.

617. The number of ways of dividing $3n$ different things into three groups of n each when the order of groups is important =$\left[\frac{(2n)!}{(n!)^2}\right]$

618. The number of ways of dividing 3n different things into three groups of n each when the order of groups is not important =$\frac{(3n)!}{(n!)^3} \frac{1}{3!}$ the division by 3! indicates that since 3 groups have equal number of things, so the three groups are indistinguishable.

619. The number of ways of dividing 3n different things into three groups of n each when the order of groups is important =$\left[\frac{(3n)!}{(n!)^3}\right]$

620. The number of ways in which n distinct things can be distributed to r different persons =r^n

Distribution/Division of Identical Objects Among Individuals/Groups

621. The number of ways of dividing n identical items among r persons (where, 0 ≤ r ≤ n), each one of whom can receive any number (0, 1, 2,..., n) of items $= n+r-1C_{r-1}$
622. The number of ways of dividing n identical items among r persons (where, 0 ≤ r ≤ n), each one of whom must receive at least one item $= n-1C_{r-1}$
623. The number of ways in which n identical items can be divided into r groups so that no group contains less than m items and more than k items (where m < k) is coefficient of x^n in the expansion of $(x^m + x^{m+1} + \ldots\ldots + x^k)^r$, for every $m \le x_i \le k$
624. The number of ways in which n identical items can be divided into r groups such that the group x_i contains minimum a_i and maximum b_i items (where $a_i \le b_i$) is coefficient of x^n in

$$(x^{a_1} + x^{a_1+1} + \ldots\ldots + x^{b_1})(x^{a_2} + x^{a_2+1} + \ldots\ldots + x^{b_2}) \ldots\ldots (x^{a_r} + x^{a_r+1} + \ldots\ldots + x^{b_r})$$

Algebraic Properties

For a natural number n, the given equation is $\boldsymbol{x_1 + x_2 + x_3 + \ldots + x_r = n}$

625. If $x_i \ge 0$, the number of integral solutions is $n+r-1C_{r-1}$
626. If $x_i \ge 1$, the number of integral solutions is $n-1C_{r-1}$
627. If $m \le x_i \le k$, the number integral solutions is coefficient of x^n in the expansion of $(x^m + x^{m+1} + \ldots + x^k)^r$
628. If $a_1 \le x_1 \le b_1, a_2 \le x_2 \le b_2 \ldots\ldots, a_r \le x_r \le b_r$ the number of integral solutions is coefficient of x^nin $\left(x^{a_1} + x^{a_1+1} + \ldots + x^{b_1}\right)\left(x^{a_2} + x^{a_2+1} + \ldots\ldots + x^{b_2}\right) \ldots\ldots (x^{a_r} + x^{a_r+1} + \ldots + x^{b_r})$

For a natural number n, if the given inequation is $x_1 + x_2 + x_3 + \ldots\ldots + x_r \le n$. Then, add a dummy variable x_{r+1} in the given equation, so that inequality can be converted into equality before finding the number of solutions.

629. If $x_i \ge 0$, the number of integral solutions to $x_1 + x_2 + x_3 + \ldots\ldots + x_r \le n$ is same as the number of integral solutions of $x_1 + x_2 + x_3 + \ldots\ldots + x_r + x_{r+1} = n$. Thus the required number of integral solutions is $n+rC_r$.

For a natural number n, number of solutions of $|x_1| + |x_2| + \ldots + |x_m| = n$.

630. The total number of integral solutions of $|a| + |b| = n$ is 4n
631. The total number of integral solutions of $|a| + |b| + |c| = n$ is $4n^2 + 2$
632. The total number of integral solutions of $|a| + |b| + |c| + |d| = n$ is $\left(\frac{8n}{3}\right)(n^2 + 2)$

For a natural number n, number of terms of binomial and multinomial expressions.

633. Total number of terms in $(x+y)^n = n+1$
634. Total number of terms in $(a_1 + a_2 + \ldots a_n)^m = {}^{m+n-1}C_{n-1}$
635. Total number of terms in $(1 + x + x^2 + \ldots x^n)^m$ is (m. n) + 1
636. Binomial coefficient $nC_m = \binom{n}{m} = \frac{n!}{m!(n-m)!}$

Re-arrangement or Derangement: Rearrangement of objects such that none of the objects occupies its original place.

637. The number of ways in which exactly r letters can be placed in wrongly addressed envelopes when n letters are placed in n addressed envelopes $= {}^nP_r\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots + (-1)^r \frac{1}{r!}\right]$

638. The number of ways in which n different letters can be placed in their n addressed envelopes so that all the letters are in the wrong envelopes $= n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots\ldots+(-1)^r\frac{1}{r!}\right]$

Geometrical Properties: In a plane if there are n points of which no three are collinear, then

639. The number of straight lines that can be formed by joining them = nC_2
640. The number of triangles that can be formed by joining them =nC_3
641. The number of quadrilaterals that can be formed by joining them = nC_4
642. The number of polygons with k sides that can be formed by joining them = nC_k
643. The number of diagonals in a polygon of n sides = $nC_2 - n$

In a plane if there are n points out of which m points are collinear, then

644. The number of straight lines that can be formed by joining them $={}^n C_2 - {}^m C_2 + 1$
645. The number of triangles that can be formed by joining them$={}^n C_3 - {}^m C_3$
646. The number of polygons with k sides that can be formed by joining them = $={}^n C_k - {}^m C_k$

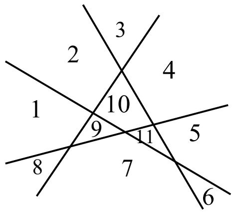
If n points are given on the circumference of the circle, then

647. The number of straight lines that can be formed by joining them =nC_2
648. The number of triangles that can be formed by joining them = nC_3
649. The number of quadrilaterals that can be formed by joining them = nC_4
650. The number of polygons with k sides that can be formed by joining them = nC_k
651. The number of diagonals in a polygon of n-sides = $nC_2 - n$
652. **The number of regions in which a line/plane/space/ sphere can be divided by n points/lines/circles/ ellipses**

Division by	Division of	Maximum Number of Parts/Regions
n Points	Line (1- Dimensional)	$\sum_{k=0}^{1} nC_k ={}^n C_0+{}^nC_1$
n Lines	Plane (2 – Dimensional)	$\sum_{k=0}^{2} nC_k ={}^n C_0+{}^nC_1+{}^nC_2$
n Planes	Solid (3 – Dimensional)	$\sum_{k=0}^{3} nC_k ={}^n C_0+{}^nC_1+{}^nC_2+{}^nC_3$
n Hyper Planes	Space (4 – Dimensional)	$\sum_{k=0}^{3} nC_k ={}^n C_0+{}^nC_1+{}^nC_2+{}^nC_3+{}^nC_4$

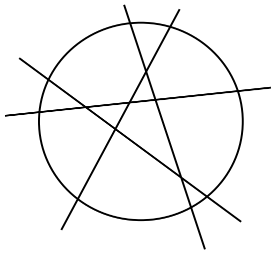
653. If n straight lines are drawn in the plane such that no two lines are parallel and no three lines are concurrent, then the maximum number of parts/regions into which these lines divide the plane is

$$nC_0+{}^nC_1+{}^nC_2 = \frac{n(n+1)}{2}+1 = \frac{n^2+n+2}{2}$$

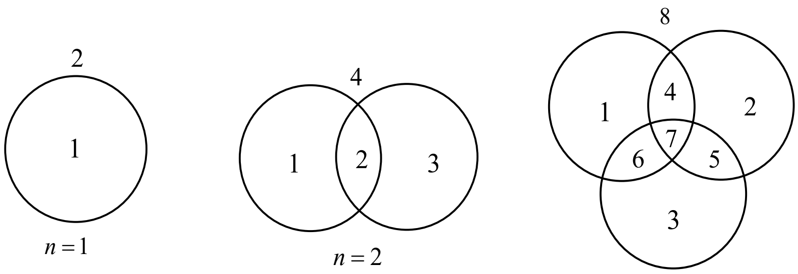


654. If n straight lines are intersecting a circle such that no two lines are parallel and no three lines are concurrent, the maximum number of parts/regions into which these lines divide the circle is

$$nC_0 + {}^nC_1 + {}^nC_2 = \frac{n(n+1)}{2} + 1 = \frac{n^2+n+2}{2}$$



655. If n circles are drawn in the plane such that no two of them are tangent, none of them lies entirely within or outside of another one and no three of them are concurrent, the maximum number of parts/regions into which these circles divide the plane is $n(n - 1) + 2$
656. If there are n overlapping triangles, the maximum number of regions into which these triangles divide the plane is $3n^2 - 3n + 2$
657. If n circles are intersecting each other, maximum number of points of intersection = $2 \times (nC_2) = {}^nP_2 = n(n-1)$
658. If n points in the plane are connected by straight lines in all the possible ways, and of these no two are coincident or parallel and no three of them are concurrent except at the given points, the number of points of intersection other than the given points of the lines so formed is $\frac{1}{8} \times \frac{n!}{(n-4)!}$



Number of Squares/Rectangles/Quadrilaterals

659. Total number of squares in a square having n columns and n rows = $= 1^2 + 2^2 + 3^2 + \dots + n^2 = \sum n^2 = \frac{n(n+1)(2n+1)}{6}$
660. Total number of rectangles in a square having n columns and n rows

$$= 1^3 + 2^3 + 3^3 + \dots + n^3 = \sum n^2 = \left(\frac{n(n+1)}{2}\right)^2$$

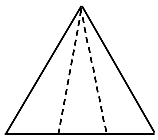
661. Total number of squares in a rectangle having m columns and n rows $= m.n + (m-1)(n-1) + (m-2)(n-2) + \ldots\ldots + 0$

662. Total number of rectangles in a rectangle having m columns and n rows

$$= (1+2+3+\ldots\ldots+m)(1+2+3+\ldots\ldots+n)$$

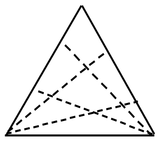
663. Total number of quadrilaterals if m parallel lines intersect other n parallel lines = $mC_2 \times^n C_2$

664. **Number of Triangles in any Triangle**: If you draw m lines from a vertex, these new lines will divide the side of the original triangle in m +1 units (or parts).

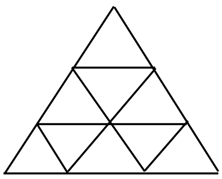


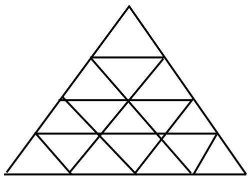
665. When lines are drawn from only one vertex If a side of a triangle is divided into n units, the total number of triangles = $\frac{n(n+1)}{2}$

666. When lines are drawn from any two vertices If each side of a triangle is divided into n units, the total number of triangles = n^3



667. When the original triangle is an equilateral triangle and lines are drawn parallel to the sides of original triangle, as shown in the following figures.





If each side of a triangle is divided into n units –

668. Total number of triangles $= \left[\frac{n(n+2)(2n+1)}{8}\right]$

Or, total number of triangles $= \frac{(-1)^n + 4n^3 + 10n^2 + 4n - 1}{16}$

Or, total number of triangles $\begin{cases} \frac{n(n+2)(2n-1)}{8}, \text{when} n \text{iseven} \\ \frac{[n(n+2)(2n+1)-1]}{8}, \text{when} n \text{isodd} \end{cases}$

669. Total number of triangles pointing upward $= \frac{n(n+1)(n+2)}{6}$

670. Total number of triangles pointing downward $= \left[\frac{n(n+2)(2n-1)}{24}\right]$

Or, total no. of triangles pointing downward $\begin{cases} \frac{1}{24}(n)(n+2)(2n-1), \text{when} n \text{iseven} \\ \frac{1}{24}(n^2-1)(2n+3), \text{when} n \text{isodd} \end{cases}$

Number of Triangles, Diagonals and Intersections in a Regular Polygon: Number of triangles formed by the vertices of a regular polygon of n sides.

671. Number of triangles formed by joining the 3 vertices of n - sided polygon $N =^n C_3 = \frac{n(n-1)(n-2)}{6}; \forall n \geq 3$

Probability

672. Probability of an event $P(A) = \frac{m}{n}$,where m is the possible positive outcomes, n is the total number of possible outcomes.

673. Range of probability values $0 \leq P(A) \leq 1$
674. Certain event P(A) = 1
675. Impossible event P(A) = 0
676. Complement $P(\overline{A}) = 1 - P(A)$
677. Independent events $P(A/B) = P(A)$, $P(B/A) = P(B)$only if A & B are mutually

    exclusive
678. Addition rule for independent events $P(A \cup B) = P(A) + P(B)$
679. Multiplication rule for independent events$P(A \cap B) = P(A).P(B)$
680. General addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
681. where $A \cup B$is the union of events A and B,
682. $A \cap B$ is the intersection of events A and B.
683. Conditional probability $P(A/B) = \frac{P(A \cap B)}{P(B)}$
684. $P(A \cap B) = P(B).P(A/B) = P(A).P(B/A)$
685. Law of total probability $P(A) = \sum_{i=1}^{m} P(B_i)P(A/B_i)$ where B_i is a sequence of mutual exclusive events.
686. Bayes' theorem$P(B/A) = \frac{P(A/B)P(B)}{P(A)}$

Set Theory & Venn Diagrams

Results on Subsets

687. The empty set φ is a subset of every set.
688. Every set is a subset of itself.
689. The number of all possible subsets of a set containing n elements is 2^n.
690. The number of all proper subsets of a set containing n elements is $(2^n - 1)$.
691. The set of all subsets of a given set A is called the power set of A, denoted by P (A). Thus, if A has n elements P (A) has 2^n elements.

Important Results on Operations on Numbers

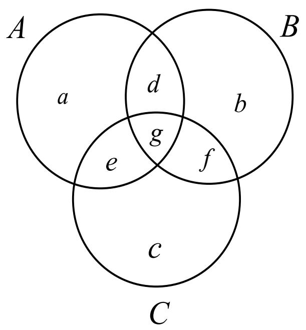
692. $A - B = A \cap B'$
693. $B - A = B \cap A'$
694. $A - B = A \Leftrightarrow A \cap B = \varphi$
695. $(A - B) \cup B = A \cup B$
696. $(A - B) \cap B = \varphi$

697. $A \subseteq B \Leftrightarrow B' \subseteq A'$
698. $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Important Results on Number of Elements in Sets

If A B, and C are finite sets, and U be the finite universal set then

699. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
700. $n(A \cup B) = n(A) + n(B) \Leftrightarrow A, B$are disjoint non-void sets.
701. $n(A - B) = n(A) - n(A \cap B)$ i.e.,
702. $n(A - B) + n(A \cap B) = n(A)$
703. $n(A\Delta B)$ =Number of elements which belong to exactly one of A or B = n (A) + n (B) – 2n $(A \cap B)$
704. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
705. number of elements in exactly two of the test A, B, C = $n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
706. number of elements in exactly one of the sets A, B, C $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$
707. $n(A' \cup B') = n[(A \cap B)'] = n(U) - n(A \cap B)$
708. $n(A' \cap B') = n[(A \cup B)'] = n(U) - n(A \cup B)$
709. **Ven Diagrams Containing Three Circles**



Let A, B, C be three books, then

a → number of people reading only A

b → number of people reading only B

c → number of people reading only C

d → number of people reading A and B both but not C

f → number of people reading B and C both but not A

e → number of people reading A and C both but not B

g → number of people reading all the three books

d + g → number of people reading A and B both

f + g → number of people reading B and C both e + g → number of people reading A and C both

a + d + g + e → number of people reading A

b + d + g + f → number of people reading B

c + f + g + e → number of people reading C

a + d + b → number of people reading A or B but not C

b + f + c → number of people reading B or C but not A

c + e + a → number of people reading A or C but not B.

a + b + c → number of people reading only one book

d + f + e → number of people reading only two books

g → number of people reading all of three books

(a + b + c) + (d + f + e) + (g) → number of people reading at least one book
(d + f + e) + (g) number of people reading at least two books
(a + b + c) + (e + f + d) + (g) → total number of readers.

710. **Finding the Values of Components**

A + B + C = (a + d + g + e) + (b + d + g + f) + (c + f + g + d)

A + B + C = (a + b + c) +2 (d + f+ e) + 3 g

A + B + C = $\alpha = 2\beta = 3\gamma$

Where $\alpha = a + b + c, \beta = d + f + e, \gamma = g$

Again $(d + g) + (f + g) + (e + g) = (d + f + e) + 3k = \beta + 3\gamma$

$\therefore (\alpha + 2\beta + 3\gamma) - (\beta + 3\gamma) = \alpha + \beta$ and $(\alpha + \beta + \gamma) - (\alpha + \beta) = \gamma$

Matrix

711. **MATRIX:** A matrix is an ordered rectangular array of the numbers or functions. The numbers of functions are called the elements of the matrix.
712. **ORDER OF MATRIX:** A matrix having m rows and n columns is called the matrix of order m × n.
713. **Row Matrix:** A matrix is said to be row matrix if it has only one row.

Ex. A = $[123]_{1\times3}$

714. **Column Matrix:** A matrix is said to be column matrix if it has only one column.

Ex. A = $\begin{bmatrix}1\\2\\3\end{bmatrix}_{3\times1}$

715. **Square Matrix:** A matrix in which the no. of rows are equal to no. of columns, is said to be square matrix.
716. **Null or (Zero) Matrix:** A matrix having all the elements zero is called zero matrix.
717. **Diagonal Matrix:** A square matrix is called diagonal matrix if all its non-diagonal elements are zero. The diagonal elements may or may not be zero.

Ex. $\begin{bmatrix}K_1 & 0\\0 & K_2\end{bmatrix}\begin{bmatrix}K_1 & 0 & 0\\0 & K_2 & 0\\0 & 0 & K_3\end{bmatrix}$

718. **Scalar Matrix:** A diagonal matrix in which all the diagonal element are equal and all other elements equal to zero is called a scalar matrix. i.e. In a scalar matrix a_{ij} = k for i = i a_{ij} = 0 for i $\neq$j. Hence $\begin{bmatrix}K & 0 & 0\\0 & K & 0\\0 & 0 & K\end{bmatrix}$is a scalar matrix.
719. **Unit Matrix or Identity Matrix:** A diagonal matrix in which all its diagonal elements are equal to 1 and all other elements equal to zero is called a unit matrix or an identity matrix.

E.g. a unit or identity matrix off order 2 and 3 are $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$and $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ respectively.

720. **Symmetric Matrix:** A square matrix A is said to be symmetric if A' = A where A' = Transpose of matrix A
721. **Skew-Symmetric Matrix:** A square matrix A is said to be skew-symmetric if A' = - A
722. **Upper Triangular Matrix:** A square matrix A=[a_{ij}] is called an upper triangular matrix a_{ij} = 0 for all i > j Thus in an upper triangular matrix, all elements below the main diagonal are zero.

e.g., $\begin{bmatrix}1 & 2 & 4 & 3\\0 & 5 & 1 & 3\\0 & 0 & 2 & 9\\0 & 0 & 0 & 5\end{bmatrix}$is an upper triangular matrix.

723. **Lower Triangular Matrix:** A square matrix A = [a_{ij}] is called a lower triangular matrix if a_{ij} = 0 for all I < j Thus in a lower triangular matrix, all elements above the main diagonal are zero.

e.g., $A = \begin{bmatrix}2 & 0 & 0\\3 & 2 & 0\\4 & 5 & 3\end{bmatrix}$ is a lower triangular matrix.

In triangular matrices (both upper and lower) non-zero elements can also be zero.

EQUALITY OF MATRICES: Two matrices A and B are said to be equal, written as A = B if

724. they are of the same order i.e. have the same number of rows and columns, and
725. each element of A is equal to the corresponding element of B that is a_{ij} = b_{ij} for all i & j

e. g. if $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then we say A = B

726. **TRANSPOSE OF A MATRIX:** If A = [a_{ij}] be an m × n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. It is denoted by A' or A^T.

Properties of transpose of the matrices-

727. (A + B)' = A' + B'
728. (KA)' = KA', where K is scalar multiple
729. (AB)' = B'A'
730. (A')' = A
731. **INVERSE OF A MATRIX:** A square matrix A of order n is invertible if there exists a square matrix B of the same order such that $AB = I_n = BA$ If matrix A is invertible, then its inverse exists. AB = I = BA, then matrix B is called inverse of matrix

A. Inverse of matrix A is denoted by A^{-1} . Hence A^{-1} = B

Note that (i) Every Invertible matrix possesses a unique inverse.

(ii) If A is an invertible matrix, then, $A^{-1}A = AA^{-1} = I$

SYMMETRIC & SKEW SYMMETRIC MATRICES

732. **Symmetric Matrix:** A square matrix A = [a_{ij}] is called a symmetric matrix, if $a_{ij} = a_{ji}$ for all I, j or $A^T = A$
733. For any square matrix A with real number entries, (A + A^T) is a symmetric matrix & (A - A^T) is a skew symmetric matrix.
734. Any square matrix A can be expressed as the sum of a symmetric & a skew symmetric matrix as $A = \left[\frac{1}{2}(A + A^T)\right] + \left[\frac{1}{2}(A - A^T)\right]$
735. **DETERMINANT:** To every square matrix A = [a_{ij}] of order n we can associate a number (real or complex) called determinant of A. It is denoted by det (A) or $|A|$

If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written a $|A| = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Note: that only square matrices have determinants.

736. **Determinant of a Matrix of Order Three:**

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$be a matrix of order 3 × 3. Then $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$ $a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$ [Expanding along first row]

= $a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$

Note that to evaluate a determinant of order three, the determinant can be expended along any row or column.

Properties of Adjoint Matrix: If A, B are square matrices of order n and $I_n$is corresponding unit matrix, then

737. A (adj. A) = $|A|I_n = (adjA)A$
738. $|adjA| = |A|_n^{-1}$
739. adj (adj A) = $|A|_n^{-2}A$
740. $|adj(adjA)| = |A|^{(n-1)^2}$
741. adj$(A^T) = (adjA)^T$
742. adj (AB) = (adj B) (adj A)
743. adj $(A_m) = (adjA)_m, m \in N$
744. adj $(kA) = k_n^{-1}(adj.A), k \in R$
745. adj$(I_n) = I_n$
746. adj O = O
747. A is symmetric ⇒adj A is also symmetric
748. A is diagonal matrix ⇒ adj A is also diagonal matrix
749. A is triangular matrix ⇒ adj A is also triangular matrix
750. A is singular matrix ⇒ $|adjA| = 0$

SOME SPECIAL CASES OF MATRIX

751. **Orthogonal Matrix**

A square matrix A is called orthogonal, if

$$AA^T = I = A^TA$$

i.e., if $A^{-1} = A^T$

Ex. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a orthogonal matrix because here $A^{-1} = A^T$

752. **Idempotent Matrix**

A square matrix A is called an idempotent matrix if $A^2 = A$

Ex. $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is a idempotent matrix because here $A^2 = A$

753. **Involutory Matrix**

A square matrix A is called an involutory matrix if $A^2 = I$or $A^{-1} = A$

Ex. A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is an involutory matrix.

754. **Nilpotent Matrix**

A square matrix A is called a nilpotent matrix if there exist p ∈N such that $A^P = 0$

Ex. $A = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ is a nilpotent matrix

755. **The Conjugate of a Matrix**

The conjugate of a matrix A whose each element is a conjugate complete number of corresponding element of matrix A.

Formula number 756:

BELIEVE IN YOURSELF